Acceleration Simulation of a Vehicle with a Continuously Variable Power Split Transmission

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(ABSTRACT)

A continuously variable transmission system has often been considered for automobiles. It offers the potential to allow the engine to operate at peak efficiency without disturbing the driver with discrete shifts. The application of CVTs in automobiles has been attempted for many years. This concept has been rapidly developed in the last twenty years. The shaft-to-shaft belt CVT is now the most commonly used CVT product in the automobiles. The main drawback of this kind of CVT is limited torque capacity and the modest power efficiency. This prevents the belt CVT from being used in the vehicles with large displacement engines.

A new concept involves a power split function. A continuously variable power split transmission (CVPST) is created by combining a V-belt CVT with a planetary gear train. The V-belt CVT is used as a control unit. By using this technology, the power flowing through the belt at low speeds is less than 50 percent. A step-up gearbox is used to expand the CVPST ratio for the applications in automobiles. The CVPST enhances the transmission torque capacity and improves the overall transmission efficiency.

This thesis involves the study of the CVPST system. Based on the analysis of the vehicle dynamics and the CVPST system, a computer program is developed. By using this program, the CVPST system design can be accomplished. The vehicle acceleration can be simulated to evaluate the CVPST performance. The acceleration simulation of vehicles equipped with standard (manual) and automatic transmissions is also possible to be carried out.
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Frontal area of the vehicle, ( m^2 )</td>
</tr>
<tr>
<td>( a_x )</td>
<td>Acceleration of the vehicle, ( m/s^2 )</td>
</tr>
<tr>
<td>( a, b, c )</td>
<td>Coefficients defined with specific engine characteristics</td>
</tr>
<tr>
<td>( C_D )</td>
<td>Aerodynamic drag coefficient</td>
</tr>
<tr>
<td>( D_A )</td>
<td>Aerodynamic drag force, N</td>
</tr>
<tr>
<td>( D_0 )</td>
<td>Initial vehicle displacement when shifting, m</td>
</tr>
<tr>
<td>( f_r )</td>
<td>Rolling resistance coefficient, for this simulation</td>
</tr>
<tr>
<td>( I_e )</td>
<td>Engine rotational inertia, N-m-sec(^2)</td>
</tr>
<tr>
<td>( I_t )</td>
<td>Transmission rotational inertia, N-m-sec(^2)</td>
</tr>
<tr>
<td>( I_d )</td>
<td>Driveshaft rotational inertia, N-m-sec(^2)</td>
</tr>
<tr>
<td>( I_w )</td>
<td>Wheels and axles shaft rotational inertia, N-m-sec(^2)</td>
</tr>
<tr>
<td>( M )</td>
<td>Mass of the vehicle, kg</td>
</tr>
<tr>
<td>( M_r )</td>
<td>Equivalent mass of the rotating components, kg</td>
</tr>
<tr>
<td>( r )</td>
<td>Radius of the wheel, m</td>
</tr>
<tr>
<td>( r_c )</td>
<td>CVT ratio</td>
</tr>
<tr>
<td>( r_f )</td>
<td>Ratio of the final drive</td>
</tr>
<tr>
<td>( r_g )</td>
<td>Ratio of the sun gear to ring gear</td>
</tr>
<tr>
<td>( r_{cg} )</td>
<td>Ratio of the counter-shaft gear to control gear</td>
</tr>
<tr>
<td>( r_o )</td>
<td>Radius of the counter-shaft gear</td>
</tr>
<tr>
<td>( r_{po} )</td>
<td>Radius of the primary pulley of the CVT, inch</td>
</tr>
<tr>
<td>( r_{ps} )</td>
<td>Radius of the secondary pulley of the CVT, inch</td>
</tr>
<tr>
<td>( r_{ri} )</td>
<td>Radius of the ring gear, inch</td>
</tr>
<tr>
<td>( r_{ro} )</td>
<td>Radius of the control gear, inch</td>
</tr>
<tr>
<td>( r_s )</td>
<td>Radius of the sun gear, inch</td>
</tr>
<tr>
<td>( r_{st} )</td>
<td>Ratio of the step-up gear</td>
</tr>
<tr>
<td>( r_{tf} )</td>
<td>Ratio of the transmission and the final drive, ( r_{tf} = r_{to} r_f )</td>
</tr>
<tr>
<td>( r_t )</td>
<td>Ratio of the transmission</td>
</tr>
<tr>
<td>( r_{to} )</td>
<td>Ratio of transmission and the step-up gear, ( r_{to} = r_t r_{st} )</td>
</tr>
<tr>
<td>( R_{hx} )</td>
<td>Hitch force of the x direction, N</td>
</tr>
<tr>
<td>( R_x )</td>
<td>Rolling resistance force, N</td>
</tr>
<tr>
<td>( R_{xf} )</td>
<td>Rolling resistance of the front wheels, N</td>
</tr>
<tr>
<td>( R_{xr} )</td>
<td>Rolling resistance of the rear wheels, N</td>
</tr>
<tr>
<td>( t )</td>
<td>Time of Vehicle acceleration, second</td>
</tr>
<tr>
<td>( t_0 )</td>
<td>Start time of acceleration, second</td>
</tr>
<tr>
<td>( T_e )</td>
<td>Torque of the engine, N-m</td>
</tr>
<tr>
<td>( T_{emax} )</td>
<td>Maximum engine torque, N-m</td>
</tr>
<tr>
<td>( T_{e0} )</td>
<td>Engine torque at idle speed, N-m</td>
</tr>
<tr>
<td>( \Delta T_e )</td>
<td>( T_{emax} - T_{e0} )</td>
</tr>
<tr>
<td>( T_r )</td>
<td>Air temperature in degrees Fahrenheit</td>
</tr>
<tr>
<td>( V )</td>
<td>Vehicle velocity; km/h</td>
</tr>
<tr>
<td>( V_0 )</td>
<td>Vehicle initial velocity when shifting, km/h</td>
</tr>
<tr>
<td>( V_W )</td>
<td>Wind speed, km/h</td>
</tr>
<tr>
<td>( V_r )</td>
<td>Vehicle velocity relative to the wind, km/h</td>
</tr>
</tbody>
</table>
W  Weight of the vehicle, kg-m/sec²

Greek Letters

\(\omega_e\)  Engine speed, rpm  
\(\omega_{e0}\)  Engine idle speed, rpm  
\(\omega_{emax}\)  Maximum engine operating speed, rpm  
\(\nabla\omega_e\)  \(\omega_{emax} - \omega_{e0}\)  
\(\eta_t\)  Efficiency of the transmission  
\(\eta\)  Total efficiency of the transmission and final drive  
\(\rho\)  Air density, N-sec²/m⁴  
\(\theta\)  Grade angle, degree  
\(\gamma\)  Circulating power ratio
CHAPTER 1 INTRODUCTION

A continuously variable transmission (CVT) provides a continuously variable ratio between the power source of a vehicle and its wheels. CVT offers the potential to allow the engine to operate at peak efficiency without disturbing the customer with discrete shifts. For many years, CVTs have been considered by many as the next step in the evolution of the automatic transmission. Incorporating CVTs in automobiles has been attempted for many years. Almost every generation of automobile engineers has tried to equip automobiles with CVT.

A century ago a rubber V-belt transmission was used in Benz and Daimler gasoline-engine-powered vehicles. A very important development of CVT was the so-called Variomatic in 1965 (Hendriks et. al., 1988). One million Volvo cars have been equipped with this kind of transmission since the year 1975. The capacity of such a CVT is limited by the power capacity of the weakest part, in this case, its rubber belt.

To enhance the CVT’s capacity, efficiency, and durability, many different kinds of CVT have been developed in the past twenty years. Of all CVT’s, the metal pushing block V-belt (MVB) CVT is the most important development. It has received very good compactness values or power density values and higher power efficiency figures than other types of traction, hydraulic, or electric CVTs (Hendriks et al., 1988).

MVB CVT has been equipped more than one million Japanese and European cars since its development (Ashley, 1994). Of all the applications of CVTs in automobiles, 99 percent are found in Japan and Europe. Only about 1 percent is found in U.S. market. These U.S. cars include the Honda Civic Hx and the Subaru Justy. The reason CVT is not popularly used in the U.S. is that torque capacity limitations have restricted the
production of CVT applications to vehicles with very small displacement engines, 1.6 \textit{l} or less, while the top three selling vehicles in this country all have displacements of 3.0 \textit{l} or greater.

The present shaft-to-shaft CVT does not offer enough torque capacity. The modest power efficiency of variable speed units, as compared to straight gears, is another main reason that the CVT is not widely used (Guebeli \textit{et. al.}, 1993). Under dynamic running conditions with a very low load at 35 km/h, the shaft-to-shaft CVT’s efficiency is only 74 percent, while at the higher load of 90 km/h it reaches 90 percent (Bonthron, 1985).

In order to take advantage of the belt CVT and overcome its disadvantages, the belt CVT conjugated with a gear train was invented (Lemmens, 1972, 1974, Takayama \textit{et al.}, 1991). The newest development is the continuously variable power split transmission (CVPST) which combines a planetary gear train with a V-belt CVT (Cowan, 1992, Mucino \textit{et al.}, 1995, 1997). Two prototypes of CVPST were presented by CK Engineering at TOPTEC-98 SAE Conference in Southfield, Michigan.

The prototype of CVPST with a parallel step-up gearbox is shown in Figure 1-1. Different from the shaft-to-shaft CVT, the input power flowing through the belt in CVPST is controlled. A suitable design may lead to the power flowing through the belt to be less than 50 percent at low speeds, while at high speeds the belt might carry as high as 90 percent of the input power. In addition, the power split function improves the total transmission efficiency, increasing the total efficiency of the transmission from 74 percent at low speeds to 85 percent. At high speeds the transmission efficiency may be as
high as 93 percent. The power split function expands the application opportunity of CVTs, enabling the CVT to be incorporated in vehicles with large displacement engines.

The CVT currently used in small engine vehicles employs a metal pushing belt. This kind of belt involves about 300 blocks and ten layers of bands. It is difficult to manufacture and is thus very expensive (Hendriks et al., 1988). CVPST also promises to use a rubber belt to replace the metal pushing belt for vehicles with small engines and further to reduce the cost of the transmission.

CVPST for automotive applications was studied by Mucino et al. (1997). There are several key parameters involved in the design of the CVPST. For a given vehicle and a given engine, it is possible to choose suitable parameters for the CVPST to allow the input power to split and flow through the gear and the belt as desired. Currently, there is

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**Figure 1-1** A prototype of CVPST with a step-up gearbox (CK Eng., Canada).

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not a simulation package available for the acceleration simulation of a vehicle with a CVPST. Based on the analysis of vehicle dynamics and the CVPST system, this thesis work involves the acceleration performance simulation of vehicles equipped with a CVPST. A computer program is developed to accomplish the CVPST system design based on certain requirements, such as the power capacity of the belt and the overall ratio of the transmission. In order to evaluate CVPST driveability, the computer program can then complete the drive test and simulation of the designed CVPST for given engine characteristics and a specific vehicle body. Using this package, acceleration simulations of vehicles with standard or automatic transmissions can also be carried out and the acceleration performance of different transmissions can be compared.
2.1 Continuously Variable Transmission Introduction

Transmission systems used in vehicles can be grouped into two main types: step and stepless ratio transmissions. The traditional multi-ratio transmissions, standard and automatic, are both step ratio transmissions and have been widely familiar to people. The continuously variable speed transmission (CVT) is relatively less familiar. Since the first generation of CVT was used in the automotive industry in 1886, CVT development never stopped. In the past twenty years, the development of the CVT includes all-gear, hydraulic, V-chains, flat belt and V-belt, and some other types of CVTs. According to the operating principles, CVT systems can be categorized into six main types as shown in Figure 2-1.

![Figure 2-1 Classification of continuously variable transmissions](image-url)
2.1.1 All Gear CVT

An all gear CVT provides the continuous variable speed ratio by gear trains. This mechanism is capable of producing a wide range of speed outputs, including very high torque at low speed and very high speed at low torque. Cook (1975) invented an all gear CVT which was different from many prior automatic transmissions requiring complex fluid logic control. It is controlled by the input and output torque. The output shaft is driven through both the input and intermediate shafts. A power source such as the automobile engine is connected through a clutch to the transmission input shaft. A load such as the automobile’s drive shaft is connected to the output shaft. At low engine speeds, it provides the highest gear ratio. When the output torque reduces to a certain level, all drive power is provided by the planetary gear train. Won (1989) disclosed another all gear CVT completing gear ratio change by a combination of floating gearing and differential gearing.

Epilogic, Inc. developed a fully geared CVT with a continuous variable ratio range of 1:1 to 1:4 (Fitz and Pires, 1991). The target application was electric vehicles. The ratio adjustment can be controlled with a simple, servo-driven actuator. The ratio adjustment varies linearly with the control actuator displacement.

2.1.2 Traction Type CVT

Continuously variable traction roller transmissions include a continuously variable transmission unit having an input disc, an output disc, and a pair of traction rollers. The traction rollers come in frictional contact with the two discs and perform control of a gear ratio by altering the contact state between the two discs and the traction rollers. An extremely high contact force allows the rollers to transmit considerable power
without slippage. Tilting the rollers changes the drive ratio between the disks. The drive is capable of an efficiency of over 98% at full forward and 80% in full reverse with power ratings of several hundred horsepower (Hibi, 1993).

The earlier traction roller type CVT is difficult to adjust and screen the loading nut at the same time in order to obtain a predetermined clearance between the disc member. In addition, a preload of the disc spring is difficult to set to a constant value due to susceptible variations in adjusting operation. A new patent (Hibi, 1993) solved these problems, however, the high cost of the required high strength materials limits its application. Continuous operation of the traction type transmission at a constant speed ratio often leads to wear of the toroidal discs and subsequent control difficulties.

### 2.1.3 Toroidal Type CVT

A toroidal type CVT (Tanaka and Imanishi, 1994) consists of an input disk, output disk and power rollers. The input disk is attached to an input shaft while the output disk is fixed to an end of the output shaft. Trunnions are swung around respective pivot axes transverse to the input shaft and the output shaft and are mounted on a support bracket arranged on an inner surface of a casing or in the casing in which the toroidal type CVT is housed. Power rollers are held between the input disk and the output disk. A loading cam-type pressure device is provided between the input shaft and the input disk to resiliently urge the input disk to the output disk. The pressure device comprises a cam plate which rotates with the input shaft and a plurality of rollers held by a holder. The rollers are rotatable around an axis which is radial to the center of the input shaft. When the cam plate is rotated with the rotation of input shaft, the rollers are urged to the outer cam surface of the input disk and the input disk is urged to the power rollers. The input
disk is rotated by the engagement of the pair of cam surfaces and the rollers. The rotation of the input disk is transmitted to the output disk through the power rollers, so that the output shaft fixed to the output disk is rotated. The ratio change is completed by swing trunnions to incline a displaceable shaft so that the power rollers abut against the different portion of the concave surfaces of the input and output shafts.

2.1.4 Hydraulic CVT

Hydraulic CVTs provide a continuously variable speed ratio by adjusting the hydraulic motor speed. This involves the adjustment of the amount of hydraulic fluid in the closed circuit. For a hydraulic CVT, either the hydraulic pump or the hydraulic motor or both, are of the variable displacement type (Kawahara et al., 1990, Inoue, 1990, Furnmoto et al., 1990). The speed ratio of this kind of CVTs is continuously variable from forward to full reverse. The general large size requirement, high noise, low efficiency and cost make this type of CVT unsuitable for automotive applications (Kumm, 1991).

2.1.5 Metal-V-Chain (MVC) CVT

Common in all designs for MVC CVT is a set of fixed sheaves with opposing movable sheaves, one for the input and one for the output shafts (Avramidis, 1986). The power is transmitted from input to output shaft by a flexible steel chain. The continuously variable speed ratio results from moving one set of sheaves together while allowing the other set of sheaves to be forced apart, thus forcing the steel chain radially outward in one sheave while inward in the other sheave. This changes the pitch diameters of the two sheaves and results in stepless ratio change. The torque transmitting mechanism is friction. Hence, the chain is actually a chain-type metal belt.
2.1.6 Flat Belt CVT

The Flat Belt CVT comprises two rotary disk assemblies. One of the assemblies is driven with an input shaft, and the other drives an output shaft to which varying loads may be applied. The torque is transmitted from the input shaft to the output shaft by an endless flat belt. There are contact pads located within slots in each of the disk assemblies. The contact pads form two circles on which the flat belt rides. The continuously variable diameter with respect to the center of each disk assembly produces a continuously variable speed ratio (Kumm, 1986).

Figure 2-2  V-belt type CVT (Hanses, 1997)

2.1.7 V-Belt CVT

The V-Belt CVT involves two pulleys and an endless V-belt as shown in Figure 2-2. Each pulley comprises two conical discs. One of the discs of at least one of the two pulleys is axially movable by means of a mechanic device (such as a spring) or a
hydraulic control cylinder. The continuously variable transmission ratio is available due to the motion of the axially movable disc(s), which changes the radii on which the belt rides. The simplest V-belt CVT is like the one introduced by Hendriks (1991). Most recent developments of V-belt CVTs comprise a gear train. V-belt CVTs can be divided into two groups according to whether the belt drive is subjected to the total input power. The V-belt CVTs, in which the input power of the transmission totally passes through (Moroto et al., 1986, Kawamoto, 1985, Robbins, 1993), usually have limited torque transferring capacity. They are used for low power situations. To enhance the belt capacity, Metal-V-Belt (MVB) (Hendriks et al., 1988) and dry hybrid-V-belt (HVB) (Fujii et al., 1992, Yuki et al., 1995,) have been developed.

In a dry hybrid V-belt a pair of tension members similar to cogged belts is inserted from both sides of lateral H-shaped blocks as shown in Figure 2-3 (a). The block is made of a special heat-resistant resin that is reinforced with Aramid fibers and carbon fibers, then molded over a high-stress aluminum frame. The tension members are fabricated in such a way that the core Aramid fibers are held between the special heat-resistant elastomer layers and coated with a heat resistant canvas. The applied torque can be 55-85 N-m.

![Figure 2-3](image)

Figure 2-3 Hybrid V-belt and metal pushing V-belt (Fujii et al, 1992, 1993)
The MVB was developed by Van Doorne’s Transmissie B. V. (VDT) in the 1980s. A complete belt unit consists of about 300 blocks connected by two sets of 10 steel bands inserted in the two sides of the metal blocks as shown in Figure 2-3 (b). The multiple layers of the steel bands provide pretension forces, which correctly guides the elements along the straight path between the two pulleys. Torque is transmitted from the drive pulley to the driven pulley by compression between the blocks. Because it attains very good compactness or power density values, MVB is the sole belt used for shaft-to-shaft CVTs at present in vehicle applications. The standard metal pushing belt is being applied in an engine range from 550 cc to 1.2 liter (Hendriks et. al., 1988). It is working well for small cars. However, the metal pushing belt is very expensive due to the difficulty in manufacturing. Also, the capacity of the belt is still not sufficient for the shaft-to-shaft CVTs to be used for vehicles with large displacement engines.

2.2 Performance of Metal V-Belt Continuously Variable Transmission

The study of the performance of CVTs with MVB began early in the 1980’s. It has been found that at the wider ratio range the CVT indicates an improvement in fuel economy. Due to the use of a CVT, vehicle power can be controlled along an ideal engine torque-speed line, which gives the best fuel consumption and emission characteristics for all power demands of the vehicle. Thus, the engine efficiency could be improved by an average of 30 percent over the city cycle and by 30 to 40 percent on highway cruise (Guo et. al., 1988). Ishibash et al. (1989) reported that the optimized combination of the two-stroke cycle engine and the CVT with other improvements enables fuel consumption to be reduced by approximately 30 percent. However, if the CVT efficiency drops 5 percent, this advantage is lost (Chana, 1986).
There were a few studies on the efficiency of the V-belt CVTs. Palmer and Bear (1977) measured the efficiency of a CVT under various operating conditions such as the variation in speeds, power outputs and speed ratios. It was found that the average CVT efficiency was about 80 to 90 percent. A similar experiment was conducted by Bent (1981), in which a relation between the axial force and the transmission efficiency was determined. Under the dynamic operation condition, Bonthron (1985) found that the V-belt CVT efficiency at low speeds was only 74 percent. The overall efficiency, speed loss and torque loss of the CVT were measured by Chen et al. (1995). The results show that the efficiency increases with an increase in external loads and is independent of the speed variation.

2.3 Power Split Technology

Lemmens (1972) described combining a planetary gear train with a V-belt CVT to produce a transmission. The input shaft is rotated by the power source. The input shaft rotates drive pulleys of both the V-belt and the chain drive. The chain drive transmits the rotation to the planetary carrier, while the V-belt transmits to the sun gear. The output shaft of the transmission is the ring gear. The majority of the power is carried by the planetary gear train, while the continuously variable V-belt drive is used to control the speed of the sun gear to gain a neutral setting and an infinitely variable range of both forward and reverse speeds. The power flowing through the V-belt is thus a reduced reaction and is kept low. The same inventor improved his invention (Lemmens, 1974) to provide a continuously variable fully automatic transmission by which the only manual setting required is the choice among neutral, forward and reverse. This arrangement may lead to power recirculation, and therefore it is not a real power split system.
Takayama et. al. (1989) presented a power split transmission which consists of a V-belt type CVT and a two-way differential clutch. The power from the engine is transmitted to the input shaft of the transmission. Two paths are operable to transmit the power from the input shaft to the output shaft. The main power transmitting path is the belt while the sub path is through the two-way differential clutch. When the CVT is near the maximum ratio, the rotating speed of the output gear in the two-way differential clutch becomes slower than that of the output gear. As a result, the output pulley shaft obtains the power from the sub power path in addition to the power of the main path. It becomes possible to increase driving power, and thus the acceleration force, and also to decrease the load carried by the belt. Using this construction, the overall transmission capacity is increased and the weight is decreased because of reducing the numbers of clutches. Because the belt carries the most input power, this proposed arrangement is less than optimum.

The combination of a flat belt CVT and a normal planetary mechanism was presented in 1991 (Kumm, 1991). This system enhances the capacity and the efficiency of the flat belt CVT and provides the reverse capability. In the low speed mode, the input power is divided into two paths: the first path through the planetary gearing to the output and a second path from the planetary gearing through the CVT back to the input shaft. In the high speed mode, a clutch on the shafting from the planetary ring gear to the output shaft is disengaged and another clutch is engaged. This permits the input power to be transmitted directly through the CVT to the output shaft. Reverse output speeds are made available by changing the radius ratio control direction in the CVT when in the low speed
mode without actuating the clutches. This arrangement eliminates the capacity limits of the belt, but the torque transmitted by the gear train is greater than the input torque.

A variable speed transmission unit (any type) connected with a planetary gear train was proposed by Cowan (1992, 1993). According to his invention, the sun gear and the primary variator are mounted on the input shaft. The output shaft is connected to the planetary carrier. There is a counter-shaft on which the secondary variator is mounted. A counter-shaft gear is meshed with a control gear which is connected to the ring gear. When the control gear is meshed with the counter-shaft gear through one or any odd number of intermediate gears and/or high torque belts or chains (allowing the ring gear and the counter-shaft gear to rotate in the same direction), the input power is split into two paths. Part of the power flows through the variators then through the counter-shaft gear to the ring gear. The rest goes directly through the sun gear. As a result, less power is transmitted by the variator system. If a V-belt CVT is used as the variable speed transmission unit, it is possible to achieve a function such that for a large input power only a fraction of the total input power load is transmitted through the belt. This proposal also exhibits the possibility that the function is achieved with a limited variator ratio range, which enables the most mechanically efficient ratio range to be selected.

2.4 Vehicle Drive Simulation

The fuel economy of automobiles equipped with hydromechanical transmissions was studied by simulation (Orshansky et. al., 1974). The simulation results showed that significant improvements in fuel economy were obtainable by using a CVT to allow the engine to operate close to best conditions. A program was developed to simulate the powertrain for fuel economy and performance of a 5 ton truck equipped with a diesel
engine in 1988 (Phillips et. al., 1990). This program was further developed for general purpose vehicle powertrain simulation. It can be used for vehicles equipped with standard and automatic transmissions. A new development is the vehicle acceleration simulation program appearing in the web site (Bowling, 1997). This program can be executed from the internet to simulate the acceleration performance of a vehicle with an automatic transmission. Due to the restriction of the software, it only can use linear torque curve for simulation. Currently, there is not a package developed for the acceleration simulation of a vehicle equipped with a continuously variable power split transmission.

2.5 Problem Description and Thesis Objective

CVTs are not widely accepted by the US market. Part of the reason is that CVTs in production today do not exploit their theoretical fuel-saving potential (Schwab, 1990). The problem may include a ratio rate of change lower than required, an optimized control not achieved and an unconventionally heavier transmission (Thompson, 1992). In fact, the most important point is that the capacity of the shaft-to-shaft CVTs is too low and the overall efficiency of this kind of CVT needs improvement. Increasing the size of the belt and transmission will diminish current CVT’s packaging advantages. The newly developed CVTs need to provide greater capacity, higher efficiency and better response (Nakano, 1992).

A V-belt CVT combined with a planetary gear train (PGT) provides a power split technology as described by Cowan (1992, 1993) and Mucino et al. (1995, 1997) and produces a continuously variable power split transmission (CVPST) system. This technology makes it possible to design a system in which the power flowing through the belt unit is controlled. Only part of the total input power is transmitted by the belt at low
speeds, while at high speeds the belt may transmit most of the input power. The output torque obtained through the planetary output shaft is greater than the torque circulating through the pulleys. This feature accomplishes two things: first the gear sizes required to attain variable velocity ratios need not change due to the CVT connection between the gear train and the output shaft; second, the belt capacity no longer limits the maximum torque capacity of the system. Hence, the CVPST may enhance the transmission capacity to enable the V-belt CVT to be used for vehicles with large displacement engines. For a small vehicle, the CVPST makes it possible to use a regular rubber V-belt as a substitute for the heavy and expensive metal pushing belt. The CVPST can also improve the total transmission efficiency because only part of the power flows through the belt at low speeds and the most efficient variator ratio range is selected. CVPST combined with a step-up gearbox may expand the overall transmission ratio to a practically useful range for the applications in automobiles. It promises to keep V-belt CVT’s advantages and to overcome its drawbacks.

For a given engine map and a given vehicle body, a careful design must be done to accomplish the power split function. This involves a CVPST parameter determination. A practically useful CVPST should also result in good driveability and acceleration performance. This thesis considers a CVPST system consisting of a V-belt CVT (hybrid belt or metal pushing belt) and a planetary gear train plus a step-up gearbox. The objective of this thesis work is the analysis of vehicle dynamics and the CVPST system, and to develop a computer program to predict the following:

- CVPST system design based on the given conditions such as the belt power capacity and the overall transmission ratio range;
• Dynamic drive test and acceleration simulation of a vehicle equipped with a CVPST and a multi-ratio standard or automatic transmission; and

• Evaluation of the driveability and acceleration performance of a CVPST compared to standard and automatic transmissions.
CHAPTER 3  VEHICLE DYNAMICS

A motor vehicle consists of many components distributed within its exterior envelope. For acceleration, braking, and most turning analysis, the whole vehicle is often treated as a lumped mass, except for the wheels. This lumped mass representing the body is called the sprung mass while the wheels are denoted as unsprung masses. The vehicle represented by the lumped mass is treated as a mass concentrated at its center of gravity (CG) as shown in Figure 3-1. The point mass at the CG, with appropriate rotational moments of inertia, is dynamically equivalent to the vehicle itself for all motions in which it is reasonable to assume the vehicle to be a rigid body. The motion of a vehicle hence belongs to a rigid body motion. A rigid body motion can be defined by six degrees of freedom. By SAE convention these six degrees of freedom are represented by six coordinates as follows:

x: Forward and on the longitudinal plane of symmetry

y: Lateral out the right side of the vehicle

z: Downward with respect to the vehicle

p: Roll velocity about the x axis

q: Pitch velocity about the y axis

r: Yaw velocity about x axis
Vehicle motion is usually described with the velocities (forward, lateral, vertical, roll, pitch and yaw) with respect to the vehicle fixed coordinate system, where the velocities are referenced to the earth’s fixed coordinate system. The earth’s fixed coordinate system is normally selected to coincide with the vehicle’s fixed coordinate system at the point where the maneuver is started. For the purpose of a drive test of a vehicle used for the evaluation of different transmissions, it is sufficient to consider only forward or longitudinal motion. The simulation of the vehicle in this paper is therefore focused on the x direction to test the characteristics of the CVPST in comparison to standard transmissions.

3.1 Newton’s Second Law

The fundamental law from which most vehicle analysis begins is Newton’s Second Law. For the translational system in the x direction, the sum of the external forces acting on the vehicle in this direction is equal to the product of the vehicle mass and the acceleration in the x direction, i.e.,

$$\sum F_x = Ma_x \quad (3-1)$$
For the rotational system, the sum of the torques acting on a body about a given axis is equal to the product of its rotational moment of inertia and the rotational acceleration about the axis, i.e.,

\[ \sum T = I \alpha \]  

(3-2)

3.2 Dynamic Loads of a Vehicle

The arbitrary dynamic loads acting on a vehicle in the x-z plane are illustrated in Figure 3-2. Among all these forces, \( F_{xf} \) and \( F_{xr} \) are traction forces, which push the vehicle to move forward along the x direction. The rest of the forces in the x direction are road resistance forces.

![Figure 3-2 Arbitrary forces acting on a vehicle (Gillespie, 1992)](image)

3.2.1 Traction Force

The traction force required to drive a vehicle comes from the engine. The engine torque is transferred to the wheels through the power train. The traction force can be calculated as:

\[
F_x = \frac{T_e r_f \eta}{r} \left[ (I_e + I_r) r_f^2 + I_d r_f^2 + I_w \right] \frac{a_x}{r^2}
\]

(3-3)
The first term of the right-hand side of Eq. (3-3) is the traction force produced by the engine. It is directly proportional to the engine output torque. The second term on the right-hand side of Eq. (3-3) represents the loss of tractive force due to the inertia of the engine and drive train components, including the clutch or torque converter, the gear box rotating components, differential units and so on.

The engine output torque varies with the throttle valve opening and the engine speed. For a given throttle valve opening, the engine torque varies with the engine speed. A typical torque surface of a gasoline engine is shown in Figure 3-3.

![Engine torque as a function of throttling valve opening and engine speed](generated from Fig. 4, Yamaguchi et. al., 1993)

It is noted from this figure that the engine torque is not a linear function of the engine speed for a given throttle valve opening. For different throttle valve openings the engine torque exhibits different features. In addition, the throttle valve opening is always
changing during the vehicle acceleration period. Therefore, the engine torque can not be
easily defined as some function. If the throttle valve opening is a function of the vehicle
speed determined by the control system, throttle valve opening at any engine speed is
known, thus the engine torque is given.

For a given engine, two curves are fixed. One is the maximum torque curve,
which corresponds to the wide open throttle valve position. The other is the so-called
optimum fuel economy path. These two curves for a certain engine are shown in Figure
3-4.

![Figure 3-4 V8 engine (300 in³) map (Gillespie, 1992)](image)

In order to perform a vehicle acceleration simulation, the throttle valve opening
must be scheduled for each gear set. Under such conditions, an engine torque curve may
be defined as a function of the engine speed.

Figure 3-5 gives five engine torque curves of a vehicle equipped with an
automatic transmission (thick lines). The operation speed range of the engine is
1000~3500 rpm. These five curves correspond to five transmission ratios. They are defined by first scheduling the throttle valve opening. Once such curves are obtained, acceleration simulation can be completed.

**Error! Objects cannot be created from editing field codes.**

Figure 3-5  Engine torque as a function of engine speed for five gear sets

Assume that the engine idling speed is $\omega_{e0}$, at which the engine torque is $T_{e0}$, and the maximum engine operating speed is $\omega_{emax}$, at which the engine reaches its torque $T_{ef}$. To find the vehicle acceleration, the torque curve between $T_{e0}$ and $T_{ef}$ can be approximately defined as a quadratic function of the engine speed $\omega_{e}$:

$$T_e = a\omega_e^2 + b\omega_e + c$$  \hfill (3-4)

The vehicle speed is directly proportional to the engine speed and dependent on the gear ratios of the transmission and the final differential drive. The relationship between the vehicle speed and the engine speed can be expressed as:

$$\omega_e = \frac{r_f}{r} \cdot V$$  \hfill (3-5)

Using Eq. (3-5) the engine torque can therefore be expressed as a function of the vehicle velocity:

$$T_e = (pV^2 + qV + c)$$  \hfill (3-6)

where the two coefficients are:

$$p = a \left( \frac{r_g}{r} \right)^2$$  \hfill (3-7)

and
\[ q = b \frac{r_d}{r} \]  \hspace{1cm} (3-8)

### 3.2.2 Aerodynamic Drag

As a result of the air stream interacting with the vehicle, six components of forces and moments are imposed. These components are drag and rolling moment in the longitudinal direction, sideforce and pitching moment in the lateral direction, and lift and yawing moment in the vertical direction. Among them, drag is the largest and most important aerodynamic force encountered by vehicles at normal highway speeds. It acts on the vehicle body in the direction of vehicle motion, i.e., x direction. Aerodynamic drag is a function of vehicle relative velocity, \( V_r \). It can be characterized by the equation:

\[ D_A = \frac{1}{2} \rho \ V_r^2 \ C_D A \]  \hspace{1cm} (3-9)

where \( \rho \) is the air density, \( A \) stands for the frontal area of the vehicle, while \( C_D \) is called the drag coefficient.

#### 3.2.2.1 Air Density

The air density is variable depending on temperature, pressure, and humidity conditions. Under standard conditions (59°F and 29.92 inches Hg or 15°C and 760 mm of Hg) the density \( \rho = 0.00236 \text{ lb-sec}^2/\text{ft}^4 \) (1.225 N-sec^2/m^4). Density under other conditions can be estimated for the prevailing pressure and temperature conditions by the equation:

\[ \rho = 0.00236 \left( \frac{P_r}{29.92} \cdot \frac{519}{460 + T_r} \right) \]  \hspace{1cm} (3-10)

where:

\[ P_r = \text{Atmospheric pressure in inches of mercury} \]
and \( T_r \) = Air temperature in degrees Fahrenheit,

or by the equation:

\[
\rho = 1.225 \left( \frac{P_r}{101.325} \cdot \frac{288.16}{273.16 + T_r} \right) \tag{3-11}
\]

where:

\( P_r \) = Atmospheric pressure in kiloPascale

and \( T_r \) = Air temperature in degrees Celsius.

### 3.2.2.2 Drag Coefficient

The drag coefficient varies over a broad range with different shapes, and should be determined by wind tunnel tests for each specific vehicle. For a passenger car made in the late 1980’s, the \( C_D = 0.3 \sim 0.4 \). For a pick-up truck, the \( C_D \) may be about 0.45 (Gillespie, 1992).

### 3.2.2.3 Vehicle Relative Velocity

Velocity used to evaluate aerodynamic drag is the relative velocity of the vehicle to the wind. This velocity can be expressed as a summation of the vehicle velocity and the component of wind velocity in the direction of the vehicle travel, i. e.,

\[
V_r = V + V_w \tag{3-12}
\]

where \( V_w \) is the wind speed component in the direction of vehicle travel. When the wind blows toward the vehicle a headwind is present. Wind blowing in the direction of vehicle travel is a tailwind. For headwind, \( V_w \) is positive, while for tailwind \( V_w \) is negative.

Bringing Eq. (3-12) into Eq. (3-9), the drag can be defined as:

\[
D_A = \frac{1}{2} \rho C_D A \left( V^2 + 2VV_w + V_w^2 \right) \tag{3-13}
\]
3.2.3 Rolling Resistance

Considering the vehicle as a whole, the total rolling resistance is the sum of the resistance from all the wheels. For theoretically correct calculations, the dynamic weight of the vehicle, including the effects of acceleration, trailer towing forces and the vertical component of air resistance should be used. However, the changing magnitude of the dynamic weight makes it very complicated to estimate vehicle performance without offering significant improvement in accuracy (Gillespie, 1992). Hence, vehicle static weight can be used to calculate rolling resistance in most cases. Rolling resistance is expressed as:

\[ R_x = R_{sf} + R_{ar} = f_r W \]  

(3-14)

To calculate rolling resistance, the rolling resistance coefficient must be defined. The rolling resistance coefficient, \( f_r \), is a dimensionless factor that reflects the effects of the complicated and interdependent physical properties of tire and ground. It proves difficult to establish standard conditions for the measurement of the effect of these variables, if it is possible. The factors affecting rolling resistance include tire temperature, tire inflation pressure/load, vehicle velocity, tire material and design, tire slip and so on. The multiple and interrelated factors affecting rolling resistance make it virtually impossible to devise an equation which takes all variables into account. It is found that at lower speeds the coefficient increases approximately linearly with speed. An equation was developed which includes a linear speed dependence (Gillespie, 1992):

\[ f_r = 0.01 \left( 1 + \frac{V}{160} \right) \]  

(3-15)

where \( V \) is the velocity in km/h.
For a broader speed range, an equation for rolling resistance was developed for rolling on concrete surface:

\[ f_r = f_o + 3.24 f_s \left( \frac{V}{160} \right)^{2.5} \]  

(3-16)

where \( f_o \) is called basic coefficient and \( f_s \) is speed effect coefficient. These two coefficients can be defined by using Figure 3-6.

At the most elementary level, the rolling resistance coefficient may be estimated as a constant. Some typical values of the rolling resistance coefficient are listed in Table 3-1.

Table 3-1 Some typical rolling resistance coefficient values (Gillespie, 1992).

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>Road Surface</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Concrete</td>
</tr>
<tr>
<td>Passenger cars</td>
<td>0.015</td>
</tr>
<tr>
<td>Heavy trucks</td>
<td>0.012</td>
</tr>
<tr>
<td>Tractor</td>
<td>0.02</td>
</tr>
</tbody>
</table>
3.3 Vehicle Acceleration

Knowing the traction force and each term of the road load, it is possible to predict the acceleration performance of the vehicle. By using the Second Law, the equation takes the form:

\[ Ma_x = F_x - R_x - D_A - R_{hx} - W \sin \theta \quad (3-17) \]

Equation (3-17) shows that the traction force term includes the engine torque and rotational inertia terms. The rotational inertias are often lumped in with the mass of the vehicle to obtain a simplified equation:

\[ (M + M_r) a_x = \frac{T_c r_f \eta}{r} - R_x - D_A - R_{hx} - W \sin \theta \quad (3-18) \]

where \( M_r \) is called the equivalent mass of the rotating components. The combination of the two masses is then called an effective mass. Let

\[ k = \frac{M + M_r}{M} \quad (3-19) \]

be the mass factor. The mass factor will depend on the operating gear. A representative number is often taken as (Gillespie, 1992):

\[ k = 1 + 0.04 + 0.0025 r_f^2 \quad (3-20) \]

Using the mass factor \( k \), Eq. (3-18) can be rewritten as

\[ kMa_x = \frac{T_c r_f \eta}{r} - R_x - D_A - R_{hx} - W \sin \theta \quad (3-21) \]

The right-hand side of Eq. (3-21) involves the vehicle velocity (first and second order), while the left-hand side involves vehicle’s acceleration, which is the first derivative of velocity to time. When CVPST is used, both \( r_f \) and \( \eta \) are functions of the
velocity. Hence a general differential equation is encountered to solve for vehicle acceleration performance. This equation can be expressed as follows:

\[ V^\prime + A(t) V^2 + B(t) V + C(t) = 0 \]  

(3-22)

Functions \( A(t) \), \( B(t) \), and \( C(t) \) depend on the torque curve, the rolling resistance equation, and the transmission selection. For a standard transmission, the total ratio and efficiency are fixed values at each gear set; while for a CVPST, they vary with the velocity (or time). These functions will be defined in Chapter 5.
CHAPTER 4 CONTINUOUSLY VARIABLE POWER SPLIT TRANSMISSION (CVPST) ANALYSIS

4.1 Model of Study

The transmission system for the study consists of a belt type CVT and a planetary gear train (PGT). The CVT combined with the PGT will produce a continuously variable transmission. Depending upon the arrangement of the CVT and PGT, the system can be grouped into Input-coupled and Output-coupled types as illustrated in Figure 4-1 (a) and (b). The possible power flows in each type of systems are shown in Figure 4-2 and 4-3. The power flow shown in Figure 4-2(a) and 4-3(a) produces power split function. The input power is shared and delivered by both the belt CVT and the PGT. The power flow shown in Figure 4-2(b), (c) and 4-3(b), (c) leads to power recirculation. A power split function makes it possible to expand the belt type CVT’s application to heavy-duty automobiles equipped with large displacement engines. This function also offers the possibility of using a rubber belt to replace the metal pushing belt used for vehicles with small engines.

Figure 4-1 Arrangement of CVT and PGT
Hsieh and Yan (1990) studied the mechanical efficiency of these two arrangements. It is found that the input-coupled type arrangement has higher mechanical efficiency. Besides this, in the input-coupled type arrangement the CVT is located at the high-speed side, the belt tension is significantly smaller; therefore, the same belt has a larger power transmitting capacity. Hence, an input-coupled system with power split function is the target CVPST model for study in the thesis. The whole system for an automobile application also includes an external step-up gearbox, which expands the CVPST ratio to a practically useful range.
4.2 System Analysis

4.2.1 CVPST Ratio

The CVPST can be formed in several different configurations. The basic system of a CVPST is schematically shown in Figure 4-4. For a planetary gear train containing the sun, planetary and internal ring gears, the radii of any two of them are independent. The sun gear and the ring gear can be chosen to define the configuration of the planetary gear train. There are two more gears involved in the definition of the functional ratios: the control gear and the counter-shaft gear. The radii of these two gears are also independent. Hence, the characteristic parameters used for defining the overall geometric configuration are the diameters (or radii) of four gears, the sun, internal ring, control and countershaft gears, and the pitch radii of the variable pulleys. Their radii (or number of teeth) are used to define the following ratios:

![Figure 4-4  A basic CVPST system (Mucino et al, 1997)](image-url)
For the planetary gear train of the CVPST, the sun gear, the ring gear and the carrier have the same rotation direction. The tangential velocities of point A, B, C, and D, shown in Figure 4-5 have the following relationships:

\[
\omega_s r_s = \omega_p r_p + \omega_i r_i \quad \text{(4-2)}
\]

\[
\omega_{ri} r_{ri} = \omega_c r_c - \omega_p r_p \quad \text{(4-3)}
\]

Where \(\omega\) is the angular velocity and \(r\) denotes the radius of gears, subscripts \(s, p, ri\) and \(c\) stand for the sun, planet, ring gears and the carrier, respectively. Considering that \(\omega_i\) is the input angular velocity and \(\omega_c\) is the output angular velocity, by using Eq. (4-1), (4-2) and (4-3) the overall ratio of this kind of CVPST is thus expressed as follows:
4.2.2 Power Split Factor

The key factor in distinguishing a power split system and a power recirculation system is the circulating power ratio. If $P_{\text{cir}}$ is used to stand for the circulating power and $P_c$ for the output power, the circulating power ratio is:

$$\gamma = \frac{P_{\text{cir}}}{P_c}$$  \hspace{1cm} (4-5)

If the input angular velocity of a differential planetary gear train is denoted as $\omega_a$, the output angular velocity is $\omega_c$ and the angular velocity of the element connected to the control unit is $\omega_b$ (see Figure 4-1a), then the circulating power ratio is (Mabie and Reinholtz, 1987):

$$\gamma = \frac{\omega_b(\omega_c - \omega_a)}{\omega_c(\omega_a - \omega_b)}$$  \hspace{1cm} (4-6)

or

$$\gamma = \frac{r'(1-R)}{1-r'}$$  \hspace{1cm} (4-7)

where $r' = \frac{\omega_b}{\omega_a}$ and $R = \frac{\omega_a}{\omega_c}$.

The circulating power ratio, $\gamma$, is a key factor. If $\gamma$ is positive, it means that there is power recirculating in the system. If $\gamma$ is negative, a power split system is formed.

For the configuration of the CVPST shown in Figure 4-4, $\omega_a = \omega_{in}$, $\omega_c = \omega_{out}$, $\omega_b = \omega_{r1}$. Because the ring gear and the control gear have the same angular velocity and it can be expressed in terms of input speed:
\[ \omega_i = \omega_m \frac{r_g}{r_c} \]

this gives:

\[ r' = \frac{r_g}{r_c} \]

Since \( R \) is the CVPST ratio given in Eq. (4-4), hence, for the configuration shown in Figure 4-4, it gives:

\[ \gamma = -\frac{r_g}{r_c r_g + r_{cg}} \tag{4-8} \]

It is obvious that this configuration is really a power split system because \( \gamma \) is always negative.

4.2.3 Power Flow

The input power of the CVPST is split into two parts flowing through the gear and the CVT belt, respectively. The amount of power flowing through the gear and the CVT depends upon the ratios defined by Eq. (4-1). If neglecting the mechanical efficiency of the transmission, the output power is equal to the input power. The power flowing through the CVT is thus expressed as:

\[ \frac{P_{CVT}}{P_{in}} = \frac{r_g}{r_c r_g + r_{cg}} \tag{4-9} \]

The torque going into the sun gear and the CVT belt can be expressed by the following equations (Mucino et al., 1997):

\[ \frac{T_{PGT}}{T_{in}} = \frac{r_c r_g}{r_c r_g + r_{cg}} \tag{4-10} \]
By using these equations, the power flowing through the PGT and the CVT can be calculated as follows:

\[ P_{CVT} = T_{CVT} \omega_m = \frac{T_{in}}{r_c r_g + r_{cg}} r_{cg} \omega_m \]  
(4-15)

\[ P_{PGT} = T_{PGT} \omega_m = \frac{T_{in} r_c r_g}{r_c r_g + r_{cg}} \omega_m \]  
(4-16)

Since the tension in the CVT belt is

\[ F_{belt} = \frac{T_{CVT}}{r_{ps}} = \frac{T_{in}}{r_{ps}} \frac{r_{cg}}{r_c r_g + r_{cg}} \]  
(4-17)

then the power through the CVT and the PGT can be expressed in terms of the belt tension:

\[ P_{CVT} = F_{belt} r_{ps} \omega_m \]  
(4-18)

\[ P_{PGT} = \omega_m (T_{in} - F_{belt} r_{ps}) \]  
(4-19)

For the CVPST it is desirable that at low speeds \( r_c = r_{c_{\text{max}}} \) the power flowing through the belt be less than that flowing through the PGT, but at high speeds \( r_c = r_{c_{\text{min}}} \)
the power flowing through the belt should be greater than that flowing through the PGT. These two situations are denoted by \( \frac{T_{PGT}}{T_{CVT}} \geq 1 \) and \( \frac{T_{PGT}}{T_{CVT}} \leq 1 \). From Eq. (4-14) these can be rewritten as \( r_{cg} \leq r_g r_{cmax} \) and \( r_{cg} \geq r_g r_{cmin} \). Hence, there must exist an intermediate value of \( r_c = r_{cm} \) in which case \( r_{cg} = r_{cm} r_g \) thus \( \frac{T_{PGT}}{T_{CVT}} = 1 \), where \( r_{cm} \) is defined by the following equation:

\[
r_{cm} = c_0 (r_{cmax} - r_{cmin}) + r_{cmin}
\]

(4-20)

where \( c_0 \) is introduced as the crossover coefficient which has the value between 0 and 1 and designates where the power crossover is to occur. Once the crossover coefficient is given, the ratio of \( r_g \) and \( r_{cg} \) is fixed. Hence, one design degree of freedom is eliminated by defining the crossover coefficient.

4.3 CVPST System Design

4.3.1 Determination of \( r_g \) and \( r_{cg} \)

A CVPST geometric configuration is determined by three ratios defined by Eq. (4-1). For an actual application of CVPST a total ratio span is assumed to be given and is denoted with \( r_{tmax} \) and \( r_{tmin} \). If a standard belt CVT is used, a certain variator ratio span is also fixed as \( r_{cmax} \) and \( r_{cmin} \). Using Eq.(4-2) and considering that \( r_c = r_{cmax} \) when \( r_t = r_{tmax} \) and \( r_c = r_{cmin} \) when \( r_t = r_{tmin} \), the following two equations are obtained:

\[
r_{r_{max}} = \frac{r_{cmax} (1 + r_g)}{r_c r_g + r_{cg}}
\]

(4-21)
From these two equations the gear ratios $r_g$ and $r_{cg}$ are expressed in terms of maximum and minimum transmission and in terms of the maximum and minimum variator ratio.

The solutions for $r_g$ and $r_{cg}$ are (Mucino et al., 1997):

\[
\begin{align*}
    r_g &= \frac{r_{c\max} r_{c\min} - r_{c\max} r_{c\min}}{r_{c\max} r_{c\min} (r_{c\min} - 1) - r_{c\min} r_{c\max} (r_{c\max} - 1)} \tag{4-23} \\
    r_{cg} &= \frac{r_{c\max} r_{c\min} (r_{c\min} - r_{c\min})}{r_{c\min} r_{c\max} (r_{c\max} - 1) - r_{c\max} r_{c\min} (r_{c\min} - 1)} \tag{4-24}
\end{align*}
\]

Eq. (4-23) and (4-24) are used to determine $r_g$ and $r_{cg}$ by the high and low values of the transmission and variator ratios. Knowing these two ratios, the crossover coefficient $c_0$ can be found.

If the crossover coefficient is first given, $r_g$ and $r_{cg}$ no longer are independent of each other. In this case, Eq. (4-20) is used to find $r_{cm}$ which denotes the ratio of $r_{cg}$ to $r_g$; then either the maximum transmission ratio or the minimum transmission ratio is fixed, while the other is found by solving Eq. (4-23) and (4-24) simultaneously. Assuming that the maximum transmission ratio is fixed, the minimum transmission ratio is found by:

\[
    r_{t\min} = \frac{r_{c\min}}{r_{c\max}} \frac{r_{c\min} + r_{c\max}}{r_{c\min} + r_{c\max}} r_{t\max} \tag{4-25}
\]

Once $r_{c\min}$ is found, Eq. (4-23) and (4-24) are used to find the values of $r_g$ and $r_{cg}$.

The ratio values of $r_g$ and $r_{cg}$ may be further limited by the belt torque capacity. Assume that the belt power capacity is known. If the total input torque is given, the torque transferred by the PGT is also known as $T_{PGT} = T_{in} - T_{CVT}$. In this case, the ratio of $r_{cg}$ to $r_g$ is also a fixed value which is given by:
\[
\frac{r_{cg}}{r_s} = \frac{r_c}{r_{PGT}} \quad (4-26)
\]

Then similar steps are executed to find \( r_{tmin} \) and then \( r_s \) and \( r_{cg} \).

### 4.3.2 Step-up Gear Ratio

It is possible to produce a continuously variable transmission with a wide span of overall ratios by using a step-up gearbox to expand the limited CVPST ratio span for the automotive application. This enables the variator ratio to be chosen within a narrow range in which the CVT has the maximum mechanical efficiency which improves the total efficiency of the transmission. For the design of a CVPST system, the step-up gear ratio, \( r_{st} \), needs to be determined.

It is assumed that the overall ratio of the transmission, including the CVPST and the step-up gear is denoted as \( r_{to} \). This ratio can be expressed as:

\[
r_{to} = r_tr_{st} \quad (4-27)
\]

For a given CVPST ratio span, the overall ratio is obtained by setting the step-up gear ratio. The step-up gear ratio is thus calculated by:

\[
r_{st} = \frac{r_{to}}{r_t} \quad (4-28)
\]

When the vehicle starts acceleration at low speeds, the required torque is large. To provide the vehicle enough traction force for acceleration and to split the most power flowing through the PGT, the maximum total ratio is required and the maximum CVPST ratio is used. In this case, the step-up gear ratio, denoted as \( r_{st1} \), is:

\[
r_{st1} = \frac{r_{to\text{ max}}}{r_{t\text{ max}}} \quad (4-29)
\]
When the variator ratio decreases to its minimum value during the vehicle acceleration, which means the CVPST ratio also reaches its minimum value, a shift occurs. The first step-up gear set is separated and the second step-up gear set is meshed. To maintain a continuous ratio, the second step-up gear ratio must satisfy the following relationship:

\[ r_{st2} \geq \frac{r_{st1}r_{\min}}{r_{\max}} \]  \hspace{1cm} (4-30)

Eq. (4-30) gives a lower bond for the second step-up gear ratio. When the vehicle reaches the target velocity and starts to cruise at that velocity, an ideal state is that the variator ratio becomes 1, which gives maximum transmission efficiency. In this case, the step-up gear ratio should satisfy the following relationship:

\[ r_{st2} = 3.6 \frac{\omega_{\max}r}{V_{\max}r_f r_{th}} \]

where \( r_{th} \) stands for the ratio of the CVPST with the maximum efficiency. If choosing variator ratio to be 1, \( r_{th} \) can be found as:

\[ r_{th} = \frac{1 + r_g}{r_g + r_{cg}} \]

Hence, the second step-up gear ratio is:

\[ r_{st2} = 3.6 \frac{\omega_{\max}r_g + r_{cg}}{V_{\max}r_f 1 + r_g} \] \hspace{1cm} (4-31)

Eq.(4-31) gives an upper bound for the step-up gear ratio. If overdrive is required, the step-up gear ratio should be found by the given minimum total ratio:

\[ r_{st2} = \frac{r_{to\ min}}{r_{th}} \] \hspace{1cm} (4-32)
From the analysis above, at least two step-up gear sets are necessary to construct a continuous ratio span and to accomplish the vehicle acceleration and the change from the acceleration to the cruise travel.

### 4.3.3 Design Example

Following is an example to show how to use these equations for the CVPST geometric design. It is assumed that the CVPST ratio wanted is from 2.5 (maximum ratio) to about 1.0 (minimum ratio) and that a standard CVT has the feasible ratio range from 2.5 to 0.5. The total ratio including step-up gear ratio is supposed to be 3.875 at start. If the vehicle target velocity for acceleration is 100 km/h (62.5 mph), the engine maximum operation speed is 3500 rpm, and the wheel radius is 0.3 m, find the ratios of \( r_g \), \( r_{cg} \), \( r_{st1} \) and \( r_{st2} \).

**Given:**
- \( r_t = 1\sim2.7 \)
- \( r_c = 0.5\sim2.0 \)
- \( r_{to \text{ max}} = 3.875 \)
- \( V_{max} = 100 \text{ km/h} \)
- \( \omega_{emax} = 3500 \text{ rpm} = 366.5 \text{ rad/sec} \)
- \( r = 0.3 \text{ m} \)

**Find:**
- \( r_g = ? \)
- \( r_{cg} = ? \)
- \( r_{st1} = ? \)
- \( r_{st2} = ? \)

**Solution:**

1. \( r_g \) and \( r_{cg} \)
   
   (1) Using Eq.(4-23) and (4-24)
   
   \[
   r_g = \frac{(1.0)(2.5) - (0.5)(2.5)}{(1.0)(2.5)(2.5 - 1) - (0.5)(2.5)(1.0 - 1)} = 0.33
   \]
   
   \[
   r_{cg} = \frac{(0.5)(2.5)(2.5 - 1.0)}{(1.0)(2.5)(2.5 - 1) - (0.5)(2.5)(1.0 - 1)} = 0.5
   \]

   The power crossover occurs at variator ratio \( r_{cm} \):
   
   \[
   r_{cm} = \frac{r_{cg}}{r_g} = \frac{0.5}{0.33} = 1.5
   \]
The corresponding crossover coefficient is:

\[ c_0 = \frac{r_{cm} - r_{c_{\text{min}}}}{r_{c_{\text{max}}} - r_{c_{\text{min}}}} = \frac{1.5 - 0.5}{2.5 - 0.5} = 0.5 \]

(2) Setting the crossover coefficient \( c_0 = 0.75 \), which leads to the power crossover to occur closer to the low pulley ratio, and using Eq. (4-20)

\[ r_{cm} = (0.75)(2.5-0.5) + 0.5 = 2 \]

then using Eq. (4-22)

\[ r_{c_{\text{min}}} = \frac{0.5}{2.5} \left( 2 + 2.5 \right) = \frac{2.5}{2 + 0.5} = 0.9 \]

Now \( r_g \) and \( r_{cg} \) can be found by using Eq. (4-23) and (4-24). They are:

\[ r_g = \frac{(0.9)(2.5) - (0.5)(2.5)}{(0.9)(2.5 - 1) - (0.5)(2.5)(0.9 - 1)} = 0.286 \]

\[ r_{cg} = \frac{(0.5)(2.5)(2.5 - 0.9)}{(0.9)(2.5 - 1) - (0.5)(2.5)(0.9 - 1)} = 0.572 \]

(3) If the belt torque limit is 40% of the input torque at the low speed with \( r_c = 2.5 \), then using Eq. (4-26), (4-25), (4-23) and (4-24), \( r_g \) and \( r_{cg} \) can be found by the following steps:

\[ r_{cm} = \frac{r_g}{r_g} = \frac{r_c}{T_{CVT}} = \frac{2.5}{0.4} = 1.67 \]

\[ r_{c_{\text{min}}} = \frac{0.5}{2.5} \frac{1.67 + 2.5}{1.67 + 0.5} = 0.96 \]

\[ r_g = \frac{(0.96)(2.5) - (0.5)(2.5)}{(0.96)(2.5 - 1) - (0.5)(2.5)(0.96 - 1)} = 0.315 \]

\[ r_{cg} = \frac{(0.5)(2.5)(2.5 - 0.96)}{(0.96)(2.5 - 1) - (0.5)(2.5)(0.96 - 1)} = 0.527 \]

2. Step-up Gear ratios

Using Eq. (4-29), the first step-up gear ratio is found as:
\[ r_{st1} = \frac{15.5}{(4.0)(2.5)} = 1.55 \]

If \( r_g = 0.28 \) and \( r_{cg} = 0.57 \) then \( r_{th} = r_t = 1.5 \) when \( r_c = 1.0 \), therefore

\[ r_{st2} = \frac{3.6 \times (366.5)(0.3)}{(100)(4.0)(1.5)} = 0.66 \]

Since \( r_{st1} r_{r_{min}} = (1.55)(0.9) = 1.395 < 1.65 = (0.66)(2.5) = r_{st2} r_{r_{max}} \), a continuous ratio is obtained.

If overdrive is required and the minimum total transmission ratio is 0.694, then the second step-up gear ratio should be:

\[ r_{st2} = \frac{0.694}{1.5} = 0.46 \]

Since \( r_{st2} r_{r_{max}} = (0.46)(2.5) = 1.15 < 1.395 \), in order to obtain a continuous ratio, some modification is required. Assuming that the minimum CVPST ratio is changeable, the new \( r_{min} \) is found by setting the two sides of Eq. (4-32) to be equal. This gives:

\[ r_{r_{min}} = \frac{r_{st2} r_{r_{max}}}{r_{st1}} = \frac{(0.46)(2.5)}{1.55} = 0.74 \]

By using the new \( r_{min} \), \( r_g \) and \( r_{cg} \) can be found. They are \( r_g = 0.19 \) and \( r_{cg} = 0.71 \). The corresponding power flowing through the belt is 60 percent of the input power. The belt capacity should be checked after this modification.

The step-up gear ratio selection above enables the vehicle to have a continuously variable transmission ratio span of 3.875~0.694.
CHAPTER 5 SIMULATION EQUATIONS OF VEHICLE ACCELERATION

Vehicle drive test and simulation involves the solution of the differential equation (3-22). The solution of this equation depends upon certain conditions. For vehicles equipped with a CVPST at low speeds, and for vehicles with standard transmissions or automatic transmissions neglecting the change in torque converter efficiency, the total ratio and efficiency for each gear set are constant. The engine speed may experience high-low-high variations during the acceleration period. For vehicles with CVPST at higher speeds, both the total ratio and efficiency are velocity (or time) dependent, while the engine can operate at a constant speed. Hence, the solution for Eq. (3-22) is different. Since vehicles with CVPST may encounter a fixed ratio and thus a fixed efficiency at low speeds, in this chapter Eq. (3-22) is first solved by considering both the total ratio and efficiency as constant. Then simulation equations for the CVPST are derived based on some assumptions. The solution obtained in this chapter can also be used for the simulation of vehicles equipped with step-ratio transmissions.

5.1 Simulation Equations for Constant Transmission Ratio and Efficiency

5.1.1 Equation Simplification

Using Eq. (3-6), (3-13), and (3-16), Eq. (3-22) may take the following form:

\[
kMa_x = \frac{r_g \eta \rho}{r} (pV^2 + qV + c) - \left( f_o + 3.24 f_s \left( \frac{V}{160} \right)^5 \right) W
\]

\[
- \frac{1}{2} \rho C_p A (V^2 + 2VV_w + V_w^2) - R_{hs} - W \sin \theta
\]

Obviously it is very difficult to find a theoretical solution for Eq. (5-1). If the rolling resistance coefficient is defined as a linear function of the vehicle velocity, as
using Eq. (3-15), for a fixed gear ratio and mechanical efficiency, Eq. (5-1) may take the following form:

\[
\frac{dV}{dt} = PV^2 + QV + C
\]  

(5-2)

where P, Q, and C are constant for each gear set. Eq. (5-2) is variable-separable, thus, a theoretical solution is available for a given set of values of P, Q, and C. Unfortunately, what kind of form the solution will take depends upon the values of P, Q, and C. There is not a unique solution function. For example, if the right-hand side of Eq. (5-2) is \(V^2 + 4V + 4\), the solution involves \(1/t\); If it is \(V^2 + V + 2\), the solution is related to \(\tan(t)\); while \(V^2 + 3V + 2\) may lead the solution to take the form of an exponential expression. The uncertain solution of the function makes it very difficult to program if it is even possible. To solve this equation for vehicle velocity and acceleration as functions of time, Eq. (5-1) must be simplified based on some reasonable conditions.

5.1.1.1 Engine Torque

From the description in Chapter 3, the output torque of an engine is a function of the engine speed and the throttle valve opening. For a vehicle equipped with a standard transmission, the throttle valve opening is directly controlled by the driver. Different drivers will produce different vehicle performance. For a vehicle equipped with an automatic transmission, it might now be controlled by an electronic control system. This allows the driver to select a preferred shift schedule mode by means of a control switch operation. The common thing for such multi-ratio transmissions is that the engine speed keeps varying in the vehicle acceleration process due to shifting. Different shift schedules will produce different engine operating paths, thus producing different vehicle velocity
and acceleration characteristics. To simulate the vehicle acceleration performance, the first step is to define the engine torque function.

For a given engine map, the engine torque can be defined as a function of vehicle speed as described with Eq. (3-6) in Chapter 3. To avoid the second order term, the quadratic torque curve is represented with several straight lines described as follows:

\[ T_e = qV + c \]  

(5-3)

Each straight line is applied to Eq. (3-6) as the traction force term. For different gear ratios and segments of torque curve, \( q \) and \( c \) should be calculated by using different \( r_{if} \) values and torque curve parameters.

5.1.1.2 Rolling Resistance

The rolling resistance coefficient, as mentioned in Chapter 3, can be evaluated by Eq. (3-15) or (3-16). Assume that the vehicle is accelerated from \( V = 0 \) to \( V = 120 \) km/h (75 mph). If using Eq. (3-15):

\[ f_r = 0.01 \left( 1 + (0 \sim 120) / 160 \right) = 0.01 \text{ to } 0.0175 \]

If using Eq.(3-16) and considering 30 psi inflation pressure which corresponds to \( f_o = 0.01 \) and \( f_s = 0.005 \) from Figure 3-6, then

\[ f_r = 0.01 + 3.24 \times 0.005 \times \left( (0 \sim 120) / 160 \right)^{2.5} = 0.01 \text{ to } 0.0179 \]

It is obvious that there is little difference between the results obtained by using these two equations. Linear Eq. (3-15) simplifies the rolling resistance term and has little effect on accuracy.

5.1.1.3 Drag Term

Drag term is related to the relative velocity of the vehicle. The wind speed is assumed to be a constant for the simulation. Hence, the drag will vary with the vehicle
velocity. Theoretically, the drag term should be combined with the traction force term. If the time interval of the simulation is small enough, the velocity change from step to step is not large. It is therefore possible to use an estimated velocity to evaluate the drag. To minimize the error, the vehicle velocity at the last simulation step plus the velocity difference of the last two steps is used as the estimated vehicle velocity. This enables the drag to be treated as a constant term at each step. In such a way the accuracy is sufficient.

5.1.2 Solution of the Equation for Constant Ratio and Efficiency

By representing the torque curve with several linear functions, using the linear equation for rolling resistance, and treating drag as a constant term, Eq. (5-1) becomes:

\[
kM \frac{dV}{dt} = \frac{r_f \eta}{r} (qV + c) - (0.01(1 + V / 160) + \sin \theta)W - \frac{1}{2} \rho C_D A (V_e + V_w)^2 - R_{hx}
\]

Eq. (5-4) can be simply expressed as

\[
\frac{dV}{dt} = QV + C
\]

where the coefficients \( Q \) and \( C \) are:

\[
Q = \frac{1}{kM} \left( \frac{r_f \eta}{r} q - \frac{Mg}{16000} \right)
\]

\[
C = \frac{1}{kM} \left( \frac{r_f \eta}{r} c - (0.01 + \sin \theta) W - \frac{1}{2} \rho C_D A (V_e + V_w)^2 - R_{hx} \right)
\]

\( V_e \) is the estimated velocity. When \( t = 0 \) \( V_e = 0 \); when \( t = 0.1 \) second (simulation time interval), \( V_e \) is assumed to be 0.1.

The vehicle velocity as a function of the acceleration time can be found by solving Eq. (5-5). It gives:

\[
V = 3.6 \left( \frac{C + QV_0 / 3.6}{Q} e^{Q(t - t_0)} - \frac{C}{Q} \right) \quad \text{(km/h)}
\]
The vehicle acceleration and displacement, as functions of the acceleration time, can then be obtained by finding the first derivative of Eq. (5-8) to time and integrating Eq. (5-8) to time, respectively. They are:

\[ a_x = (C + QV_0 / 3.6)e^{Q(t-t_0)} \] (m/s²) \hspace{1cm} (5-9)

\[ D = \frac{C + QV_0 / 3.6}{Q^2} (e^{Q(t-t_0)} - 1) - \frac{C}{Q} (t - t_0) + D_0 \] (m) \hspace{1cm} (5-10)

Eq. (5-8), (5-9) and (5-10) can be used for the simulation of vehicles equipped with standard and automatic transmissions, as well as the CVPST when the transmission ratio is a constant value at low vehicle speeds.

5.2 Simulation Equations for Variable Transmission Ratio and Efficiency

Vehicles equipped with CVPST present a different acceleration performance. Due to the stepless ratio change, it is possible for the engine to work at the optimum point at which it reaches its maximum efficiency (Guo et. al., 1988). For the acceleration simulation of a vehicle with CVPST combined with a step-up gearbox, it is assumed that the vehicle acceleration from the static state to a target velocity involves three stages. In the first stage, the vehicle starts to accelerate with the engine speed increasing from the idling speed to the maximum operation speed. After the engine reaches the maximum operation speed, it is possible to stay at that speed, while the transmission ratio will decrease with the increase in vehicle velocity. At the first stage, a large total ratio is necessary to ensure that the vehicle gains sufficient traction force for acceleration. The
CVPST should also distribute the majority of power flowing through the gear. From the analysis described in Chapter 4, a higher variator ratio will lead to a higher CVPST ratio. A high CVPST ratio combined with a step-up gear ratio should produce a large transmission ratio. This matches the large ratio requirement. From Eq. (4-15) it is noted that the power flowing through the belt decreases with the increase in the variator ratio. Hence, to use as high a variator ratio as possible also matches the requirement of reducing the power flowing through the belt at low speeds. It is therefore a smart choice for the CVPST ratio to be fixed at a high value at the low speed stage. Once the engine reaches its maximum operating speed, the first stage is over. In the second stage, the CVPST ratio starts decreasing with the increase in the vehicle velocity. When the CVPST ratio is decreased to its minimum value (or some designed value), the first step-up gear is separated and the second step-up gear starts to mesh. The second stage is completed and the third stage begins. The CVPST ratio should then be adjusted to a certain value to keep the total transmission ratio the same as before switching. For example, if the total transmission ratio is 3.875~0.72 and CVPST ratio is 2.7~0.9, two step-up gear ratios are 1.435 and 0.48, and step-up gear switch occurs at the CVPST ratio of 0.9; the CVPST ratio should be adjusted back to 2.7. In the third stage, the CVPST ratio keeps decreasing until the vehicle reaches the target velocity. The optimum condition is that the variator ratio of the CVPST is back to 1 when the vehicle is accelerated to the target velocity, because this enables the vehicle to cruise at that speed with the maximum transmission and engine efficiency.
5.2.1 Shift Velocity

It is assumed that the vehicle is accelerated from a static state to a certain velocity. At stage I, the engine speed increases from the idle speed to the maximum operating speed. Vehicle velocity increases with the increase in engine speed. The CVPST takes the maximum ratio and the first step-up gear set is meshed. The first shift velocity, $V_1$, is found by:

$$V_1 = 3.6 \frac{\omega_{e \text{ max}} r}{r_f r_{st1} r_{max}}$$

(5-11)

At stage II, the CVPST ratio continuously decreases from its maximum value to the minimum value. The first step-up gear is still in use. The vehicle velocity increases with the decrease in the CVPST ratio. The second shift velocity is easily found by the following equation:

$$V_2 = 3.6 \frac{\omega_{e \text{ max}} r}{r_f r_{st1} r_{min}}$$

(5-12)

At stage III, the second step-up gear is meshed and the CVPST ratio goes back to some value (corresponding to the total ratio at the end of stage II), then again decreases continuously. This value can be calculated as:

$$r_i = \frac{r_{st1}}{r_{st2}}$$

(5-13)

When the vehicle velocity reaches the target velocity, the CVPST ratio will be adjusted to $r_{th}$ which corresponds to the variator ratio of 1, the engine speed is:

$$\omega_e = \frac{V_{max} r_f r_{th} r_{st2}}{3.6 r}$$

(5-14)
5.2.2 Equations for Simulation

The vehicle acceleration at stage I is the same as that of a vehicle equipped with a standard transmission. At this stage the vehicle velocity, acceleration, and displacement can be calculated with Eq. (5-8), (5-9) and (5-10).

Different from the first stage, at stage II and III the engine keeps working at its maximum speed with a constant throttle opening, while the transmission ratio varies with a decrease in the variator ratio. Therefore, the input torque will remain a constant value, but the traction force will be a function of the transmission ratio and efficiency, which is also a function of the transmission ratio.

5.2.2.1 Transmission Ratio

At stage II and III, the transmission ratio varies with the change of variator ratio $r_c$ during the vehicle acceleration. At any acceleration time $t$, the transmission ratio (including the step-up gear ratio) has to obey the following relationship:

$$
r_{to} = \frac{\omega_{e, \text{max}}}{r_j V} \tag{5-15}
$$

where

$$
r_{to} = r_tr_{st} \tag{5-16}
$$

5.2.2.2 CVPST Efficiency

As described before, the CVT efficiency is a function of the variator ratio. The power loss of a CVPST as a function of the variator ratio is shown in Figure 5-1. This figure shows that the maximum efficiency is obtained at $r_c = 1$. Different arrangements of CVPST may produce different power loss curves, however, in most cases, the power loss curves of the CVPST have similar configurations. From this power loss curve, the CVPST efficiency as a function of the variator ratio can be found as shown in Figure 5-2.
Figure 5-1  Power loss of CVT as a function of variator ratio (by Philippe Arques)

Figure 5-2  CVPST efficiency as a function of variator ratio
The overall efficiency of the CVPST also reaches its maximum value at the variator ratio of 1.0. This CVPST efficiency curve can be best fitted by two quadratic functions. They are:

\[ \eta_i = -36.091r_c^2 + 69.84r_c + 59.47 \quad (0.5 \leq r_c \leq 1.0) \]

and

\[ \eta_i = -4.2592r_c^2 + 7.9364r_c + 89.54 \quad (1.0 \leq r_c \leq 2.0) \]

These two equations can be rewritten in a generic form as follows:

\[ \eta_i = nr_c^2 + sr_c + u \quad (5-17) \]

### 5.2.3 Differential Equation and Solutions

At the second and the third stages, there are two choices to evaluate the rolling resistance force. One is to calculate the rolling resistance force using the velocity obtained from the last step; the other is using a constant value for \( f_r \). The drag is calculated with the same method as used in the standard transmission simulation. The mass factor, \( k \), is considered a constant since the total inertia is almost the same at these stages, though the variator ratio changes. Mechanical efficiency is evaluated using the current variator ratio before each simulation step. It can be treated as a constant for each step. The differential equation for these two stages becomes:

\[ kMa_x = \frac{T_{r_{max}}r_f\eta_i}{r}r_{to} - F_{AD} - (f_r + \sin \theta)W - R_{hx} \quad (5-18) \]

Considering that the overall transmission ratio, \( r_{to} \), is a function of the vehicle velocity as defined with Eq. (5-15), the first term of the right-hand side of Eq. (5-18) can be rewritten as:
\[ \frac{T_{e\text{ max}}\omega_{e\text{ max}}\eta}{V} \]

Eq.(5-18) is simply written as follows:

\[ a_x = \frac{Q}{V} + P \]  
(5-19)

or

\[ \frac{dV}{dt} = \frac{Q}{V} + P \]  
(5-20)

where

\[ Q = \frac{T_{e\text{ max}}\omega_{e\text{ max}}\eta}{kM} \]  
(5-21)

\[ P = -\frac{1}{kM}\left(F_{AD} + (f_r + \sin \theta) W + R_{hc}\right) \]  
(5-22)

and \( F_{AD} \) is the drag and is calculated with Eq. (3-9).

By separating variables and considering the initial condition \( V = V_0 \) at \( t = t_0 \), Eq. (5-20) is solved as

\[ t = \frac{1}{P}(V - V_0) - \frac{Q}{P^2} \ln \left( \frac{Q + PV}{Q + PV_0} \right) + t_0 \]  
(5-23)

The acceleration time required for a specific target vehicle velocity is easily found by using this equation. The vehicle acceleration at any simulation step is directly calculated from the vehicle velocity using Eq. (5-19).

If neglecting the road resistance force term, Eq. (5-21) becomes:

\[ \frac{dV}{dt} = \frac{Q}{V} \]  
(5-24)

The solution of this equation gives the Power-Limited Acceleration performance of a vehicle. The vehicle velocity, acceleration, and displacement at this state are defined by following equations:
\[ V = \sqrt{2Q(t-t_0) + V_0^2} \]  \hspace{1cm} (5-25)

\[ a_s = \frac{Q}{\sqrt{2Q(t-t_0) + V_0^2}} \]  \hspace{1cm} (5-26)

\[ D = \frac{1}{3Q} \left[ (2Q(t-t_0) + V_0^2) \right] - V_0^3 \right] + D_0 \]  \hspace{1cm} (5-27)
CHAPTER 6 ACCELERATION SIMULATION RESULTS AND ANALYSIS

A computer program developed in this thesis work simulates the acceleration performance of vehicles equipped with CVPST. This program also can simulate the acceleration performance of vehicles equipped with multi-ratio transmissions (standard or automatic). Another task for this program is to choose the appropriate parameters for CVPST based on the given engine characteristics and vehicle information. The program language used is Microsoft Visual Basic 5.0. This software can provide a user-friendly interface which is suitable when many inputs need to be typed in. It is also very easy to produce the corresponding graphs using the data generated by the program itself. The program consists of three main parts, which are the CVPST system design, the acceleration simulation of vehicle equipped with the CVPST and the multi-ratio transmission. Program code of these three parts are given in Appendix A. The program is available to simulate as many as five speed standard or automatic transmissions. The CVPST system design is completed according to the given belt torque capacity or transmitted torque requirement.

6.1 Simulation Conditions

6.1.1 Simulation Model

Figure 6-1 shows a block diagram of a vehicle equipped with a typical gasoline engine and a standard transmission or an automatic transmission. The difference between these two is that for the standard transmission a clutch is used, while for the automatic transmission a torque converter is mounted. Figure 6-2 illustrates a block diagram of a vehicle equipped with a CVPST plus a step-up gearbox. Since at low speed the variator
ratio is supposed to be a fixed value, the CVPST with a step-up gearbox is similar to a 2-speed standard transmission.

Figure 6-1  Simulation model of vehicle with standard or automatic transmission

Figure 6-2  Simulation model of a CVPST with a step-up gearbox
Figure 6-3 gives the characteristics of the engine used in the simulation. The CVPST with a step-up gearbox may produce the same transmission ratio range as the standard transmission and the automatic transmission, allowing the CVPST to be compared with the other two transmissions. Each parameter related to the vehicle acceleration performance, such as the vehicle weight, the drag coefficient or other aerodynamic parameters, can possibly change. The simulation example presented in the thesis work selects the transmission ratio span of 0.694 ~ 3.875. With a differential ratio of 4.0, the total gear ratio from the engine to the wheels is from 15.5 to 2.776. The gear ratios of the transmission (both standard and automatic) are given in Table 6-1. The CVPST parameters for the simulation are also listed in Table 6-1 as a reference. The CVPST simulation, including both the low and high speeds range, is left to discuss later.

![Figure 6-3 Characteristics of the engine for the simulation (Yamaguchi et. al., 1993)](image)
Table 6-1 Transmission ratios for simulation.

<table>
<thead>
<tr>
<th>Gear</th>
<th>Gear Ratio</th>
<th>Gear</th>
<th>Gear Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.875</td>
<td>CVT</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>2.14</td>
<td>Control/Counter-Shaft</td>
<td>0.28</td>
</tr>
<tr>
<td>3</td>
<td>1.384</td>
<td>Sun/Ring</td>
<td>0.57</td>
</tr>
<tr>
<td>4</td>
<td>1.000</td>
<td>CVPST</td>
<td>2.5</td>
</tr>
<tr>
<td>5</td>
<td>0.694</td>
<td>Step-up</td>
<td>1.55</td>
</tr>
</tbody>
</table>

6.1.2 Shift Velocity

There is a direct relationship between the engine speed and the vehicle velocity as described in Chapter 3. Considering unit converting coefficients and using Eq. (3-5), the engine speed, $\omega_e$ (rpm), can be defined as a function of the vehicle velocity $V$:

$$\omega_e = V \cdot \frac{r_e \times 10^3}{2\pi r \cdot 60} = 2.6526 \frac{r_e}{r} \cdot V$$  

where the velocity and the wheel radius take the unit of kilometer per hour (km/h) and meter (m), respectively. If the total ratio for each gear set is given and the wheel radius is known, the engine speed-vehicle velocity chart can be defined. Figure 6-4 shows such a chart as defined by the parameters shown in Table 6-2.

The shift velocities of the vehicle depend upon the gear ratios and the defined engine maximum operation speed. It is assumed that the maximum engine operation speed is 3500 rpm (which can vary depending on the engine). It is also assumed that the
gearshifts are executed when the engine reaches its maximum speed. Therefore, shift velocities are obtained by setting $\omega_e = 3500$ rpm, as also shown in Table 6-2.

Table 6-2 Transmission gear ratios, differential ratio and wheel radius.

<table>
<thead>
<tr>
<th>Gear</th>
<th>Gear Ratio $r_t$</th>
<th>Differential Ratio $r_f$</th>
<th>Wheel Radius $r$ (mm)</th>
<th>$\omega_e$ (V)</th>
<th>Shift Velocity (km/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.875</td>
<td>4.0</td>
<td>279</td>
<td>147.37V</td>
<td>23.75</td>
</tr>
<tr>
<td>2</td>
<td>2.140</td>
<td></td>
<td></td>
<td>81.38V</td>
<td>43.00</td>
</tr>
<tr>
<td>3</td>
<td>1.384</td>
<td></td>
<td></td>
<td>52.63V</td>
<td>66.50</td>
</tr>
<tr>
<td>4</td>
<td>1.000</td>
<td></td>
<td></td>
<td>38.03V</td>
<td>92.03</td>
</tr>
<tr>
<td>5</td>
<td>0.694</td>
<td></td>
<td></td>
<td>26.39V</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6-4 Engine speed-vehicle velocity chart
6.1.3 Simulation Assumptions

A whole period of vehicle acceleration involves the gear shifts. Shifting gear takes a certain time. For different transmissions, the time required is different. For the CVPST, there exist a variator ratio variation from minimum value to some larger value. The time needed for this change is usually longer than that for one gear shift. Some further research work is necessary but not including in the thesis to fully understand the time required for the variator ratio change. In this thesis, it is assumed that the shift is completed in an instant for both the multi-ratio transmission and the CVPST.

6.2 Simulation of Torque–Limited Acceleration with a Standard Transmission

6.2.1 Engine Torque

For the torque-limited acceleration (TLA), it is assumed that there is no slip between the drive wheels and the road and the engine throttle is wide open. The maximum torque possibly obtained varies with the engine speed following the curve of TVO = 100%. This curve is shown in Figure 6-5. It is also assumed that the engine idling speed is 1000 rpm. The engine curve at the speed range (1000 ~ 3500 rpm) is best fitted to the following function:

\[ T_e = -0.000004\omega_e^2 + 0.0329\omega_e + 92.5 \quad (6-1) \]

Hence, the constant coefficients are: \( a = -0.000004 \); \( b = 0.0329 \); \( c = 92.5 \).
Figure 6-5 The maximum torque curve of the engine

Figure 6-6 General input data form for simulation
6.2.2 Parameters for Simulation

The parameters used for the simulation include the data related to the vehicle, road, environment and other conditions. Figure 6-6 shows a general input data form, which lists the information and the parameters required to accomplish the simulation. All parameters on this form may be modified. The data appearing on this form are used for one of the simulation examples. Figure 6-7 shows a transmission-related data form. The data are used for TLA simulation. The torque curve is defined by three coefficients $a$, $b$ and $c$.

![Standard Transmission](image)

**Figure 6-7** Multi-ratio transmission input data form

6.2.3 Simulation Results

The general simulation results for a vehicle with a 5-speed standard transmission is printed out as a form given in Figure 6-8. To achieve the target velocity of 100 km/h, 9.7 seconds acceleration is required. The vehicle velocity, acceleration and displacement
as functions of acceleration time as well as transmission ratio as a function of time are shown in Figure 6-9 (a), (b), (c) and (d).

Figure 6-8  Simulation result of a vehicle with a 5-speed transmission (TLA)
Figure 6-9 (a) Vehicle velocity as a function of time (Standard, TLA)

Figure 6-9 (b) Vehicle acceleration as a function of time (Standard, TLA)
Figure 6-9 (c) Vehicle displacement as a function of time (Standard, TLA)

Figure 6-9 (d) Transmission ratio as a function of time (Standard, TLA)
6.3 Simulation of Best Fuel Economy Acceleration with an Automatic Transmission

6.3.1 Torque Curve

With the same engine map shown in Figure 6-3, a shift schedule for the best fuel economy acceleration (BFEA) was developed by Yamaguchi et. al.(1993), as shown in Figure 6-10. With this shift schedule and the engine speed-vehicle velocity chart (Figure 6-4), the corresponding operating points to each throttle opening curve for each gear set are found in the engine map as shown in Figure 6-11. The operating points of each gear set can be connected to form an operating path of the engine at this gear ratio. Hence, the engine torque as a function of engine operating speed for each gear ratio is found.

Figure 6-10 Shift schedule for best fuel economy (Yamaguchi et. al.,1993)
The five engine torque curves can be fitted with straight lines as shown in Figure 6-11. The fitting functions are shown in Table 6-4. Once these torque curve functions are obtained, the following simulation procedure is similar to that of the simulation of the torque-limited acceleration with a standard transmission.

**6.3.2 Data for Simulation**

The general data used for this simulation is the same as that used in the torque-limited acceleration simulation. Figure 6-12 shows the form used for the user to input data related to the transmission and the torque curve parameters for each gear set.

**6.3.3 Simulation Results**

Figure 6-13 (a), (b), and (c) illustrate the corresponding vehicle velocity, acceleration, and displacement as functions of time. The general simulation results are given in Figure 6-14.
Figure 6-12  Transmission input data of an automatic transmission for BFEA

<table>
<thead>
<tr>
<th>Gear set</th>
<th>Gear Ratio</th>
<th>Shift Velocity (km/h)</th>
<th>Torque Curve Parameters at this gear set</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st gear</td>
<td>3.875</td>
<td>Uncertain</td>
<td>$T_e = a \cdot w^2 + b \cdot w + c$ (in rpm)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$a = 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b = 0.0232$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$c = 66.8$</td>
</tr>
<tr>
<td>2nd gear</td>
<td>2.14</td>
<td>Uncertain</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$0.0258$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$43.68$</td>
</tr>
<tr>
<td>3rd gear</td>
<td>1.384</td>
<td>Uncertain</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$0.0221$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$43.5$</td>
</tr>
<tr>
<td>4th gear</td>
<td>1.000</td>
<td>Uncertain</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$0.00625$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$68.125$</td>
</tr>
<tr>
<td>5th gear</td>
<td>0.694</td>
<td>100</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$0.04$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$-16$</td>
</tr>
</tbody>
</table>

Transmission Efficiency: 0.98

Figure 6-13 (a) Vehicle velocity as a function of time (Automatic, BFEA)
Figure 6-13 (b) Vehicle acceleration as a function of time (Automatic, BFEA)

Figure 6-13 (c) Vehicle displacement as a function of time (Automatic, BFEA)
6.4 Acceleration Simulation of a Vehicle with a CVPST

6.4.1 Simulation Model

The acceleration simulation of a vehicle with a CVPST is based on the given parameters of the CVPST with a step-up gearbox. The program developed for the CVPST simulation first conducts a system design of the CVPST based on some user-defined conditions. The first step provides all the necessary ratios. The example of the CVPST in this paper has the ratio range of 2.5 ~0.75. Combined with the step-up gearbox with ratios of 1.55 and 0.46, the overall transmission ratio span is 3.875 to 0.694, which is the same as that of the standard and automatic transmissions described before. Figure 6-15 shows a
block diagram of a vehicle equipped with a typical gasoline engine and a CVPST with a step-up gearbox.

![Block diagram of vehicle with CVPST](image)

**Figure 6-15 Simulation model of vehicles with a CVPST**

To compare the CVPST to the standard and automatic transmissions, the same engine curve used for TLA with the standard transmission is used for TLA with the CVPST. The torque function used for the first gear set of the automatic transmission will be used for BFEA of the vehicle with the CVPST.

**6.4.2 Simulation of Stage I**

The analysis in Chapter 4 shows that the CVPST ratio will be fixed at this stage. Therefore, the constant efficiency of the CVPST will involve in this stage. The first acceleration stage is completed when the engine speed reaches the redline speed. The simulation results obtained at this stage will be treated as the initial values of stage II.
6.4.3 Simulation of Stage II and III

Vehicle acceleration at these two stages requires the definition of the CVPST efficiency as a function of the variator ratio. The simulation procedure is first giving a velocity, then finding the time required. Vehicle acceleration is found by using the given velocity. Simulation involves the application of Eq. (5-19) and (5-23).

![Figure 6-16 Transmission input data form for CVPST](image)

The general data for the CVPST simulation are the same as before. Figure 6-16 gives one of the transmission-related forms, which is used for the torque-limited acceleration simulation. For the best fuel economy simulation, the maximum torque is the same as that used for multi-ratio automatic transmissions. Torque curve coefficient $c$ for the best fuel economy acceleration simulation is $c = 82$. The corresponding maximum torque is 148 N-m.
6.4.4 Simulation Results

The Engine Speed-Velocity Chart shown in Figure 6-17 illustrates that the CVPST works like a multi-ratio transmission with infinite gear ratios between its ratio range. Simulation results for the torque-limited acceleration are shown in Figure 6-18. The CVPST combined with a step-up gearbox with two pairs of gears forms a continuously variable transmission. The overall transmission ratio is presented by the top line going down through the middle and the bottom lines, as shown in Figure 6-19. It clearly shows that the transmission ratio is continuously decreasing from its maximum value to the minimum value. The CVPST ratio and the variator ratio are also illustrated in the same figure. At the first stage, these two ratios are constant. At Stage II and III, they vary nonlinearly with the time.
Figure 6-18  Simulation results of torque-limited acceleration (TLA) with CVPST
The vehicle velocity and acceleration curves are given in Figure 6-20 (a), and (b). For the best fuel economy acceleration, the ratio curves are given in Figure 6-21, while the velocity and the acceleration curves are shown in Figure 6-22(a) and (b).
Figure 6-20 (b) Vehicle acceleration as a function of time (CVPST, TLA)

Figure 6-21 Transmission ratio as a function of time (CVPST, BFEA)
Figure 6-22 (a) Vehicle velocity as a function of time (CVPST, BFEA)

Figure 6-22 (b) Vehicle acceleration as a function of time (CVPST, BFEA)
The power split function for the CVPST is shown in Figure 6-23. The power flowing through the belt at low speeds is about 44 percent of the input power. With the decrease in the variator ratio, the power flowing through the belt increases. When the variator ratio decreases to 0.5 the power flowing through the belt reaches 80 percent of the input power. With the shift of the step-up gear and the variator ratio goes back to 2.5, the power flowing through the belt reduces to 44% of the input power and then reincreases with the decrease in variator ratio.

![Power Split Curve](Image)

Figure 6-23 Power flowing through the belt and gear as functions of variator ratio

Using the gear ratios obtained from the system design, the gear sizes can be determined. If the diametral pitch of the planetary gear train is 4 and the diametral pitch of the step-up gear sets is 3, the gear sizes of a basic CVPST system are given in Table 6-3.
Table 6-3 Gear size example for a basic CVPST system.

<table>
<thead>
<tr>
<th>Gear</th>
<th>Number of Teeth</th>
<th>Pitch Diameter (inch)</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>28</td>
<td>7</td>
<td>Diametral Pitch $p_d = 4$</td>
</tr>
<tr>
<td>Ring</td>
<td>100</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>Counter-Shaft</td>
<td>32</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Control</td>
<td>56</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Step-up Gear Set 1</td>
<td>Drive</td>
<td>18</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Driven</td>
<td>28</td>
<td>9.33</td>
</tr>
<tr>
<td>Step-up Gear Set 2</td>
<td>Drive</td>
<td>31</td>
<td>10.33</td>
</tr>
<tr>
<td></td>
<td>Driven</td>
<td>15</td>
<td>5</td>
</tr>
</tbody>
</table>

6.5 Discussion of Results

As shown in the figures above, the CVPST presents a continuous ratio variation. The velocity curve, compared to the standard transmission, seems the same. The acceleration curve of the CVPST, except for the start stage at which the engine speeds up to its maximum operating speed, is smooth and continuous. The user does not experience a jerk, making the vehicle very comfortable to ride.

The acceleration simulation results of vehicles with the standard, automatic and CVPST transmissions are shown in Table 6-3. For the torque-limited acceleration, if the target velocity is 100 km/h (62.5 mph), under the same conditions, the vehicle with the standard transmission needs 9.7 seconds; the vehicle with the CVPST needs 7.9 seconds. The difference between them is 1.8 seconds. If the target velocity is 120 km/h (75 mph) the difference is 3.7 seconds.
Table 6-4 Simulation results of vehicles with different transmissions.

<table>
<thead>
<tr>
<th>Vehicle Velocity (km/h)</th>
<th>Acceleration Time (sec)</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CVPST</td>
<td>Standard</td>
</tr>
<tr>
<td>20</td>
<td>1.3</td>
<td>1.1</td>
</tr>
<tr>
<td>40</td>
<td>2.2</td>
<td>2.2</td>
</tr>
<tr>
<td>60</td>
<td>3.6</td>
<td>3.9</td>
</tr>
<tr>
<td>80</td>
<td>5.9</td>
<td>6.3</td>
</tr>
<tr>
<td>100</td>
<td>7.9</td>
<td>9.7</td>
</tr>
<tr>
<td>120</td>
<td>10.9</td>
<td>14.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>1.5</td>
<td>1.2</td>
</tr>
<tr>
<td>40</td>
<td>2.4</td>
<td>2.4</td>
</tr>
<tr>
<td>60</td>
<td>4.0</td>
<td>4.2</td>
</tr>
<tr>
<td>80</td>
<td>6.4</td>
<td>6.8</td>
</tr>
<tr>
<td>100</td>
<td>8.7</td>
<td>10.5</td>
</tr>
<tr>
<td>120</td>
<td>11.7</td>
<td>16</td>
</tr>
</tbody>
</table>

The maximum efficiency of the CVPST is about 93.5%; the minimum efficiency is about 85%. Compared to the shaft-to-shaft CVT, they are increased for 3.8% and 14.8%, respectively. The average mechanical efficiency of the CVPST is about 89.5%. At low speeds, the belt carries only 44% of the input power. The power shifts from the gear to the belt in a continuous manner.

By using the first torque curve of the engine used for the best fuel economy acceleration with the automatic transmission, the acceleration performance of the vehicle with the CVPST is simulated. In this case, the acceleration time is much shorter than the vehicle with the automatic transmission. If a good driver driving a vehicle with a standard transmission could follow the optimum path along which the best fuel economy is obtained, the vehicle would not obtain as good an acceleration performance as the vehicle with a CVPST.
For the best fuel economy acceleration, the engine of the vehicle with a CVPST operates at the best efficiency range, except for the start stage. Due to the shifts, the engine of a vehicle with a standard or an automatic transmission has to work following a zigzag path, thus it is impossible to stay at the optimum range. The better fuel economy for the vehicle with a CVPST is obvious.

6.6 Conclusion

The vehicle acceleration simulation displays that the CVPST can provide a continuously variable ratio change and thus a smooth and continuous acceleration change. It is more comfortable for the driver and the passengers to ride a vehicle equipped with a CVPST. The simulation of the vehicle acceleration performance presents that both the driveability and the fuel economy for the vehicle with a CVPST are better than the one with a standard or a multi-ratio automatic transmission. A power split function is available for the CVPST which expands the power capacity of the belt CVT. At low speeds, the belt only carries part of the input power. A required power split percentage is obtained by using the computer program to accomplish the system design. It is possible to shift the power flow in the system from the gear to the CVT unit in a continuous manner.

6.7 Future Recommendations

The simulation of the CVPST uses the fitted function of the power loss curve. This curve is obtained under certain conditions. The next phase of this study should pay attention to the study of power loss of the belt CVT, especially the loss at both entrance and exit of both the primary and secondary pulleys. The study should also find the inertia variation of the CVPST due to the variator ratio change. Since the variator ratio changes
nonlinearly during the acceleration, an electric control system is strongly recommended.
Another valuable work is to develop a prototype control system which meets the
operation requirement.
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APPENDIX-A PROGRAM FLOW CHART AND CODE FOR SIMULATION

1. Flow Chart of the Simulation Program
Multi-ratio transmission simulation

For i = 1 to No. of gear sets

rtf = rt(i) * rf

Find V, ax, D

i = i + dt

Yes

V > Vs(i)?

No

V > Vmax?

Yes

i = i + 1

Exit Sub
2. Main Page Code

Private Sub enterCommand_Click()
   Cls
    M = Val(wtText.Text) + Val(driverwtText.Text)
    W = M * 9.81
    power = Val(pwText.Text)
    omegaEmax = Val(maxenginespeedText.Text) * 3.1415926 / 30
    omegaE0 = Val(idleenginespeedText.Text) * 3.1415926 / 30
    Temax = Val(MaxTorqueText.Text)
    Te0 = Val(idlespeedtorqueText.Text)
    r = Val(wheelradiusText.Text) / 1000
    rf = Val(diffratioText.Text)
    eitaf = Val(differefficiencyText.Text)
    area = Val(frontareaText.Text)
    rou = Val(airdensityText.Text)
    CD = Val(dragcoefficientText.Text)
    Vw = Val(windspeedText.Text)
    fr = Val(rollingcoefficientText.Text)
    Rhx = Val(hitchforceText.Text)
    sita = Val(roadgradeText.Text)
    sita = Atn(sita / 100)
    t0 = Val(starttimeText.Text)
    V0 = Val(startvelocityText.Text)
    D0 = Val(startdisText.Text)
    t01 = t0
    V01 = V0
    D01 = D0
    Vmax = Val(maxvelocityText.Text)
    If engineinertiaText.Text = "Unknown" Or transmissioninertiaText.Text = "Unknown" Or driveshaftinertiaText.Text = "Unknown" Or wheelaxleinertiaText.Text = "Unknown" Then
dynamicmass = "unknown"
Else
    Ie = Val(engineinertiaText.Text)
    It = Val(transmissioninertiaText.Text)
    Id = Val(driveshaftinertiaText.Text)
    Iw = Val(wheelaxleinertialText.Text)
End If
If Optioncvt.Value Then
    Background.savestandardchart.Enabled = False
    Background.savegearratio.Enabled = False
    Background.viewgearratio.Enabled = False
    Background.viewchart.Enabled = False
    Background.manuprintGchart.Enabled = False
    Background.manuprintgearratio.Enabled = False
Else
    Background.savecvpstrchart.Enabled = False
    Background.savecvpstratio.Enabled = False
    Background.viewcvpstratio.Enabled = False
    Background.viewcvtchart.Enabled = False
    Background.manuprintcvtchart.Enabled = False
    Background.manuprintcvtratio.Enabled = False
End If
If Option5speedauto.Value Then
    BFEAform.Show
ElseIf Option5speedmanual.Value Then
    speed5manualform.Show
Else
    cvtform.Show
End If
End Sub
3. Code for CVPST Design and Simulation

(1) Main program

Private Sub enterCommand_Click()
  rtotalmax = Val(ratioText(1).Text)
  rtotalmin = Val(ratioText(2).Text)
  rtmax = Val(ratioText(3).Text)
  rtmin = Val(ratioText(4).Text)
  rcmax = Val(ratioText(5).Text)
  rcmin = Val(ratioText(6).Text)
  eitagear = Val(stepupgearefficiencyText.Text)
  S1 = Val(s1Text.Text) / 100
  S2 = Val(s2Text.Text) / 100
  N1 = Val(n1Text.Text) / 100
  N2 = Val(n2Text.Text) / 100
  U1 = Val(u1Text.Text) / 100
  U2 = Val(u2Text.Text) / 100
  a = Val(aText.Text) * (30 / 3.1416) ^ 2
  b = Val(bText.Text) * 30 / 3.1416
  cc = Val(cText.Text)
  TTmax = a * omegaEmax ^ 2 + b * omegaEmax + cc
  If rtotalmax <= rtotalmin Or rtmax <= rtmin Or rcmax <= rcmin Or eitagear > 1 Then
    MsgBox "You typed invalid value(s)!
    Unload Me
    Load Me"
  End If
  If belttorqueText.Text = "Unlimited" And torquepercentageText.Text = "Unlimited" _
    And crossovercoefficientText.Text = "Unknown" Then
    case1 = 0
  ElseIf belttorqueText.Text <> "Unlimited" And torquepercentageText.Text = "Unlimited" _
    And crossovercoefficientText.Text = "Unknown" Then
    case1 = 1
  ElseIf belttorqueText.Text = "Unlimited" And torquepercentageText.Text <> "Unlimited" _
    And crossovercoefficientText.Text = "Unknown" Then
    case1 = 2
  ElseIf belttorqueText.Text = "Unlimited" And torquepercentageText.Text = "Unlimited" _
    And crossovercoefficientText.Text <> "Unknown" Then
    case1 = 3
  ElseIf belttorqueText.Text <> "Unlimited" And torquepercentageText.Text <> "Unlimited" _
    And crossovercoefficientText.Text = "Unknown" Then
    case1 = 4
  ElseIf belttorqueText.Text <> "Unlimited" And torquepercentageText.Text = "Unlimited" _
    And crossovercoefficientText.Text <> "Unknown" Then
    case1 = 5
  ElseIf belttorqueText.Text = "Unlimited" And torquepercentageText.Text <> "Unlimited" _
    And crossovercoefficientText.Text <> "Unknown" Then
    case1 = 6
  Else
    case1 = 7
  End If
  If crossovercoefficientText.Text <> "Unknown" Then

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If Val(crossovercoefficientText.Text) >= 1 Or Val(crossovercoefficientText.Text) <= 0 Then
    MsgBox "Invalid crossover value!"
    Unload Me
    Me.Show
    Exit Sub
Else
    c0 = Val(crossovercoefficientText.Text)
End If
End If
If belttorqueText.Text <> "Unlimited" Then
    If Val(belttorqueText.Text) <= 0 Then
        MsgBox "Invalid belt torque value!"
        Unload Me
        Me.Show
        Exit Sub
    Else
        Tbmax = Val(belttorqueText.Text)
    End If
End If
If torquepercentageText.Text <> "Unlimited" Then
    If Val(torquepercentageText.Text) <= 0 Or Val(torquepercentageText.Text) >= 100 Then
        MsgBox "Invalid belt torque percentage value!"
        Unload Me
        Me.Show
        Exit Sub
    Else
        Tbpercent = Val(torquepercentageText.Text) / 100
    End If
End If
If case1 = 0 Then
    Call designsubSub
    rcm = rcmax * rcmin * (rtmin - rtmax) / (rtmax * rcmin - rcmax * rtmin)
    c0 = (rcm - rcmin) / (rcmax - rcmin)
    Tbpercent = rcg / (rcmax * rg + rcg)
End If
If designCheck.Value Then Call cvpstdesignSub
End Sub
/Sub CVPST Design
Sub cvpstdesignSub()
Dim rcm1 As Single
Select Case case1
Case 1
    Tcvt = Tbmax
    Tpgt = Temax - Tcvt
    rcm = rcmax * Tcvt / Tpgt
End If
If simulationCheck.Value Then
    Call calculation3Sub
    Call cvpstmusub2
End If
result2Form.Show
End Sub

(2) CVPST Design
Sub cvpstdesignSub()
Dim rcm1 As Single
Select Case case1
Case 1
    Tcvt = Tbmax
    Tpgt = Temax - Tcvt
    rcm = rcmax * Tcvt / Tpgt
End If

\[ rtmin = \frac{rcmin}{rcmax} \times (rcm + rcmax) \times rtmax \times (rcm + rcmin) \]

Call designsubSub
\[ c0 = \frac{(rcm - rcmin)}{(rcmax - rcmin)} \]
\[ Tbpercent = \frac{rcg}{(rcmax \times rg + rcg)} \]

Case 2
\[ Tcvt = Temax \times Tbpercent \]
\[ Tpgt = Temax - Tcvt \]
\[ rcm = \frac{rcmax \times Tcvt}{Tpgt} \]
\[ rtmin = \frac{rcmin}{rcmax} \times (rcm + rcmax) \times rtmax \times (rcm + rcmin) \]

Call designsubSub
\[ c0 = \frac{(rcm - rcmin)}{(rcmax - rcmin)} \]
\[ Tbpercent = \frac{rcg}{(rcmax \times rg + rcg)} \]

Case 3
\[ rcm = c0 \times (rcmax - rcmin) + rcmin \]
\[ rtmin = \frac{rcmin}{rcmax} \times (rcm + rcmax) \times rtmax \times (rcm + rcmin) \]

Call designsubSub
\[ Tbpercent = \frac{rcg}{(rcmax \times rg + rcg)} \]

Case 4
If \( \frac{Tbmax}{Temax} > Tbpercent \) Then
\[ Tcvt = Tbpercent \times Temax \]
Else
\[ Tcvt = Tbmax \]
MsgBox "The belt does not have enough torque capacity. Design will be based on the maximum belt torque capacity!"
End If
\[ Tpgt = Temax - Tcvt \]
\[ rcm = \frac{rcmax \times Tcvt}{Tpgt} \]
\[ rtmin = \frac{rcmin}{rcmax} \times (rcm + rcmax) \times rtmax \times (rcm + rcmin) \]

Call designsubSub
\[ c0 = \frac{(rcm - rcmin)}{(rcmax - rcmin)} \]
\[ Tbpercent = \frac{rcg}{(rcmax \times rg + rcg)} \]

Case 5
\[ Tcvt = Tbmax \]
\[ rcm = c0 \times (rcmax - rcmin) + rcmin \]
\[ rcm1 = \frac{rcmax \times Tcvt}{(Temax - Tcvt)} \]
If \( rcm > rcm1 \) Then
\[ rcm = rcm1 \]
MsgBox "The belt does not have enough torque capacity. Design is based on the maximum belt torque capacity!"
End If
\[ rtmin = \frac{rcmin}{rcmax} \times (rcm + rcmax) \times rtmax \times (rcm + rcmin) \]

Call designsubSub
\[ c0 = \frac{(rcm - rcmin)}{(rcmax - rcmin)} \]
\[ Tbpercent = \frac{rcg}{(rcmax \times rg + rcg)} \]

Case 6
\[ Tcvt = Tbpercent \times Temax \]
\[ Tpgt = Temax - Tcvt \]
\[ rcm = \frac{rcmax \times Tcvt}{Tpgt} \]
\[ rcm1 = c0 \times (rcmax - rcmin) + rcmin \]
If \( rcm < rcm1 \) Then
MsgBox "The design is based on the torque percentage of CVT."
Else
MsgBox "The design is based on the crossover coefficient."
\[ rcm = rcm1 \]
End If
\[ rtmin = \frac{rcmin}{rcmax} \times (rcm + rcmax) \times rtmax \times (rcm + rcmin) \]

Call designsubSub
\[ c0 = \frac{(rcm - rcmin)}{(rcmax - rcmin)} \]
\[ Tbpercent = \frac{rcg}{(rcmax \times rg + rcg)} \]

Case 7
If \( Tbmax > Tbpercent \times Temax \) Then
\[ Tcvt = Tbpercent \times Temax \]
Else
\[ Tcvt = Tbmax \]
End If
Tpgt = Temax - Tcvt
rcm = rcmax * Tcvt / Tpgt
rcml = c0 * (rcmax - rcmin) + rcmin
If rcml < rcm Then rcm = rcml
rtmin = rcm / rcmax * (rcm + rcmax) * rtmax / (rcm + rcmin)
Call designsubSub

c0 = (rcm - rcmin) / (rcmax - rcmin)
Tbpercent = rcm / (rcmax * rcm + rcm)
End Select
End Sub

Sub designsubSub()
rg = (rtmax * rcmin - rcmax * rtmin) / (rtmax * rcmin * (rtmin - 1) -
rcmax * rtmin * (rtmax - 1))
rcg = rcmax * rcmin * (rtmax - rtmin) / (rtmin * rcmax * (rtmax - 1) -
rtmax * rcmin * (rtmin - 1))
rst1 = rtotalmax / rtmax
rth = (1 + rg) / (rg + rcg)
rst2 = rtotalmin / rth
If rst2 * rtmax < rst1 * rtmin Then 'discontinuous ratio
    rst2 = rst1 * rtmin / rtmax
End If
End Sub

(3) CVPST Simulation

Sub cvpstsimusub()
t0 = t
jj = j

V0 = V
D0 = D
Vs(2) = 3.6 * omegaEmax * r / rf / rst1 / rtmin
Vs(3) = Vmax
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If V < Vs(2) Then
    rst = rst1
    tshift(2) = t - t0
    i = 2
Else
    rst = rst2
    i = 3
End If
rtotal = 3.6 * omegaEmax * r / rf / V
rtt(j) = rtotal / rst
rc = rcg * rtt(j) / (1 + rg * (1 - rtt(j)))
rt(j) = rtt(j) * rst
RCC(j) = rc
If rc <= 1 Then
    N = N1
    S = S1
    U = U1
Else
    N = N2
    S = S2
    U = U2
End If
\[
eitatotal = eitaf \times eitagear \times (N \times rc^2 + S \times rc + U)
\]
\[
Q = TT_{\text{max}} \times \omega_{\text{E}_{\text{max}}} \times eitatotal \div K \div M
\]
\[
Rx = (\text{rou} \times CD \times \text{area} \times ((V_{\text{drag}} + V_{\text{w}}) \div 3.6)^2 / 2 + (fr + \sin(sita)) \times W + Rhx)
\]
\[
P = \frac{-Rx}{K \div M}
\]
\[
t = 1 \div P \div (V - V_0) \div 3.6 - Q \div P \div P \times \log((Q + P \times V \div 3.6) \div (Q + P \times V_0 \div 3.6)) + t_0
\]
\[
ax = Q \div V \times 3.6 + P
\]
\[
nm(j) = \text{Int}(t \times 10 + 0.5) \div 10
\]
\[
nv(j) = \text{Int}(V \times 100 + 0.5) \div 100
\]
\[
'\text{dd}(j) = \text{Int}(D \times 100 + 0.5) \div 100
\]

4. Code for Multi-ratio Transmission Simulation

Private Sub enterCommand_Click()
    no_of_gearset = Val(gearsetnumberText.Text)
    a = Val(aText.Text) \times (30 / 3.1416)^2
    b = Val(bText.Text) \times 30 / 3.1416
    cc = Val(cText.Text)
    eitagear = Val(gearefficiencyText.Text)
    For i = 1 To no_of_gearset
        rt(i) = Val(ratioText(i).Text)
        axs(i) = 0
        If (rt(i) = 0 Or rt(i) = Empty) Then
            MsgBox "Invalid gear ratio or number of gear set value!"
            Unload Me
            Me.Show
            Exit Sub
        End If
        If shiftvelocityText(i).Text = "Uncertain" Then
            Vs(i) = Int((\omega_{\text{E}_{\text{max}}} \times r \div rf \div rt(i)) \times 3.6 \times 100 + 0.5) \div 100
            shiftvelocityText(i).Text = Format$(Vs(i))
        Else
            Vs(i) = Val(shiftvelocityText(i).Text)
        End If
    Next

    Vs(no_of_gearset) = V_{\text{max}}
    tshift(0) = t_0
    V = V_0
    t = t_0
    eitatotal = eitaf \times eitagear
    \text{tt}(0) = \text{Int}(t \times 10 + 0.5) \div 10
    \text{vv}(0) = \text{Int}(V \times 100 + 0.5) \div 100
    \text{aaa}(0) = \text{Int}(ax \times 100 + 0.5) \div 100
    \text{dd}(0) = \text{Int}(D \times 100 + 0.5) \div 100
    RCC(0) = \text{Int}(rc \times 100 + 0.5) \div 100
    V_{\text{drag}} = V_0
j = 1
For i = 1 To no_of_gearset

50
  rtf = rt(i) * rf

  If dynamicmass = "unknown" Then 'no inertia known
    K = 1.04 + 0.0025 * rtf * rtf
  Else
    K = (M + ((Ie + It) * rtf * rtf + Id * rf * rf + Iw) / r / r) / M
  End If

  Call calculation2Sub

  tt(j) = Int(t * 10 + 0.5) / 10
  VV(j) = Int(V * 100 + 0.5) / 100
  aaa(j) = Int(ax * 100 + 0.5) / 100
  DD(j) = Int(D * 100 + 0.5) / 100
  RCC(j) = Int(rc * 100 + 0.5) / 100

  Vdrag = 2 * V - VV(j - 1)

  If V < Vs(i) Then
    If axs(i) < ax Then axs(i) = ax
    If amax < axs(i) Then amax = axs(i)
    t = t + 0.1
    j = j + 1
    tshift(i) = t - t0
    GoTo 50
  Else
    If V < Vmax Then
      D0 = D
      t0 = t
      V0 = V
    Else
      result1Form.Show
      Exit Sub
    End If
  End If
Next
result1Form.Show

End Sub

Sub calculation2Sub()
'This sub is used for torque-limited acceleration
Dim no_of_section As Integer
Dim domegaE, omegaE1, omegaE2, omegaEcurrent, Te1, Te2, Te3, Te4, bb, ccc As Single
no_of_section = 3

domegaE = (omegaEmax - omegaE0) / no_of_section
omegaE1 = omegaE0 + domegaE
omegaE2 = omegaE0 + 2 * domegaE
omegaEcurrent = V / 3.6 * rt(i) * rf / r

If a <> 0 Then
  Te1 = a * omegaE0 ^ 2 + b * omegaE0 + cc
  Te2 = a * omegaE1 ^ 2 + b * omegaE1 + cc
Else
  If tshift(i) > 0 Then
    'torque limited, apply new force
    If omegaEcurrent < omegaE1 Then
      Te1 = a * omegaE0 ^ 2 + b * omegaE0 + cc
      omegaEcurrent = omegaE1
    Else
      Te2 = a * omegaE1 ^ 2 + b * omegaE1 + cc
      omegaEcurrent = omegaE2
    End If
  End If
End If

End Sub
Te3 = a * omegaE2 ^ 2 + b * omegaE2 + cc
Te4 = a * omegaEmax ^ 2 + b * omegaEmax + cc

If omegaEcurrent <= omegaE1 Then
    bb = (Te2 - Te1) / domegaE
    ccc = Te1 - bb * omegaE0
ElseIf omegaEcurrent < omegaE2 Then
    bb = (Te3 - Te2) / domegaE
    ccc = Te2 - bb * omegaE1
Else
    bb = (Te4 - Te3) / domegaE
    ccc = Te3 - bb * omegaE2
End If
Else
    bb = b
    ccc = cc
End If

qq = bb * rtf / r
Q = qq * rtf * eitatotal / r / K / M
C = (ccc * rtf * eitatotal / r - rou * CD * area * ((Vdrag + Vw) / 3.6) ^ 2 / 2 - (fr + Sin(sita)) * W - Rhx) / K / M

V = 3.6 * ((c + Q * V0 / 3.6) / Q * Exp(Q * (t - t0)) - c / Q)
ax = (c + Q * V0 / 3.6) * Exp(Q * (t - t0))
D = (c + Q * V0 / 3.6) / Q / Q * (Exp(Q * (t - t0)) - 1) - c / Q * (t - t0) + D0

End Sub
### APPENDIX-B SIMULATION RESULTS OF A VEHICLE WITH A CVPST

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2. Best Fuel Economy Acceleration with CVPST

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VITA

Zhijian Lu was born on May 14, 1958 in Xian, Shaanxi, the People’s Republic of China. He received his secondary education at Xian 63rd Middle and High School in his hometown. He received his degree of Bachelor of Science in Mechanical Engineering from Xian Mining Institute in January 1982. Mr. Lu started to work at Xian Design and Research Institute in January 1982. He worked as a mechanical engineer from 1982 to 1992 and worked as a director and a senior engineer from 1993 to 1994.

Mr. Lu was a visiting scholar in West Virginia University from 1995 to 1997. In the fall semester of 1996, he was admitted to the Engineering and Mineral Resources Master of Science program at the Mechanical and Aerospace Engineering Department, College of Engineering and Mineral Resources, West Virginia University, Morgantown, West Virginia. Currently, he is a candidate of Master degree.

Zhijian married Shenglan Zhang in Xian, Shaanxi, China. They have a son named Qing.